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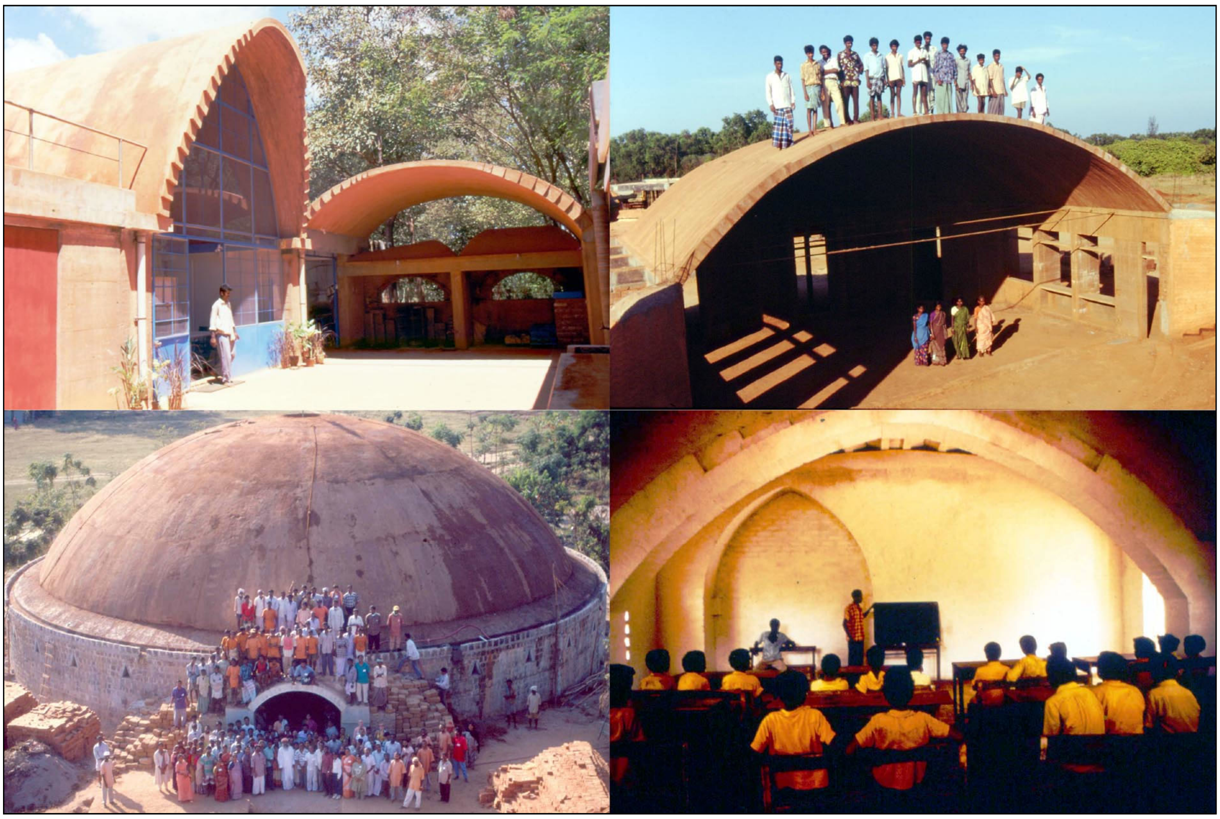
BUILDING WITH ARCHES, VAULTS AND DOMES

TRAINING MANUAL
FOR ARCHITECTS AND ENGINEERS



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TRAINING MANUAL
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127 pages

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Knowledge's wisdom

However vast is today's knowledge,
we know already that it is ignorance
compared to the knowledge of tomorrow.

Satprem

Skillful hands,
precise care,
a sustained attention,
and one compels matter to obey the spirit.

The Mother

EDITOR’S FOREWORD

Since its first publication in 2003, “Building with Arches, Vaults and Domes: Training Manual for Architects and Engineers” has served as an important users’ manual for students and practitioners alike.

The revisions since 2003 have mainly focused on optimisation methods for funicular arch analysis. However, this 2016 edition includes a number of noteworthy changes, both in structure and in content:

- Some chapters have been consolidated for ease of referencing. In particular, the “Acoustics of vaulted structures” has been included as a consolidated chapter in the annex of the manual. A separate Part has been allocated for the “Stability of Domes”, which refers to the “Stability of Arches & Vaults” where applicable.

- There have been significant chapter revisions, including: “Early history & Technical evolution”, “Basic Structural Principles”, and the introductory sections for the Catenary and Funicular methods.

- New content has been added to clarify that the structural principles outlined in this manual conform to the Limit Analysis Framework of masonry, as first codified by Jacques Heyman (Heyman, 1966).

- Structural terminology has been modified along with new terms and extended glossary referencing. For ease of referencing, each of the key terms outlined in the glossary in the back of the manual are represented in italics when the concept is first introduced in the text.

- Formulas have been revised for better clarity.

- The bibliography has been extended, with added referencing and standard formatting.

- New drawings and photographs have been added, along with a list of figures.

We hope that this work may continue to be a useful tool in the service of the wide dissemination of earthen vaulting technologies.

FOREWORD

Architects and engineers will find in this document the means to study the structural stability, design and construction of arches, vaults and domes.

The stability methods described here aim to optimise arches and vaults, in particular, the profile and thickness, in order to achieve the lightest structure for the widest span and therefore the greatest structural efficiency. In comparison with conventional approaches to equilibrium analysis in the field of structural masonry engineering, these methods offer a unique approach, which synthesizes structural analysis and construction design. While conventional graphical analysis considers only the stability of a fully built structure (and not its stability during construction or any aspects of its buildability), these methods optimise arches and vaults as an integral step in the design and construction processes.

Note that the methods used for the Optimisation Method presented here concern only arches and vaults – not domes. While methods do exist to optimise the sections of masonry domes, this will be considered as a special case. However, the basic structural principles for domes will be outlined, and the stability of domes may be safely calculated by analysing a cross-section of the dome.

Various methods of construction are described here for an explanation of how to build Arches, Vaults and Domes (AVD). This manual has been formulated specifically for masonry systems built with Compressed Stabilised Earth Blocks (CSEB), using binders of stabilised earth. For a more comprehensive description of the production of CSEB, please refer to the Earth Institute's "Production and Use of Compressed Stabilised Earth Blocks: Code of Practice".

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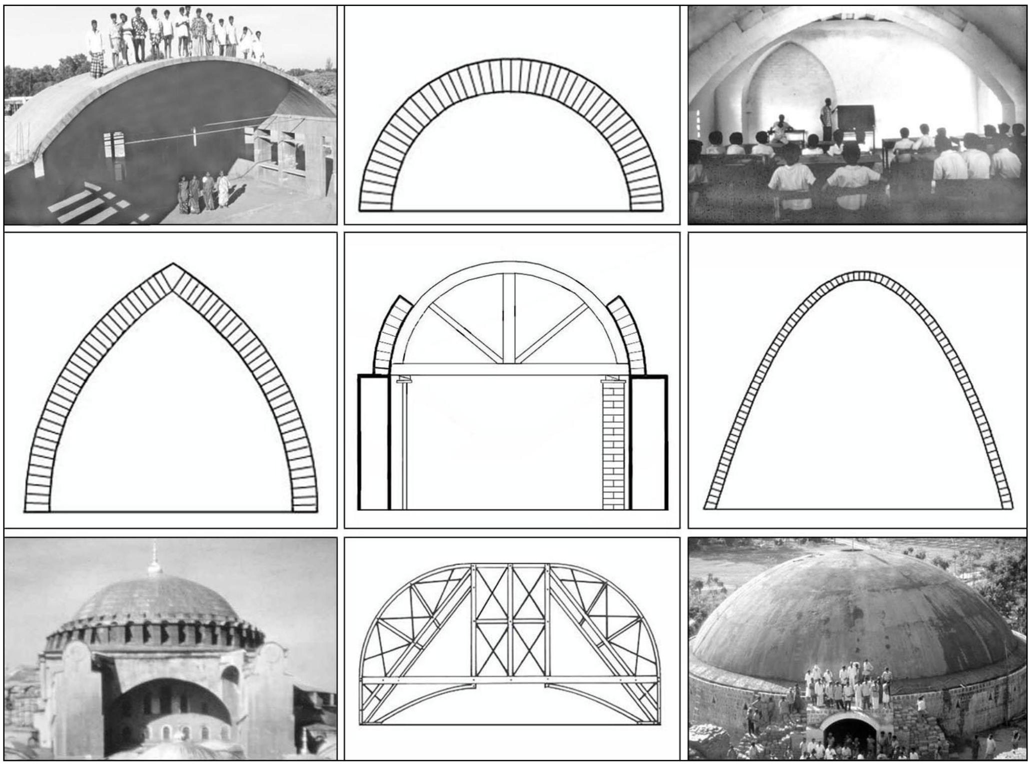


1.1 SYMBOLISM

Throughout the ages and across cultures, mankind has demonstrated a great affinity to the architecture of arches, vaults and domes. All over the world, these constructions have appeared in a vast diversity of forms according to the local context – differences which emerge from a complex matrix of environmental, technical, social and cultural patterns.

Yet, throughout all eras, the practice of arch, vault and dome construction has sprung forth from an aspiration transcending the basic needs of people to build for the purpose of dwelling. According to the extensive research of historians, the invention of the countless forms of arches, vaults and domes has been linked with the cosmologies and world views of many cultures (Smith, 1971). Whether for the purpose of primitive funereal monuments, churches, mosques, synagogues, temples, civic architecture, or even domestic residences, the arch, vault and particularly the dome have served as influential and pervasive symbols marking the ineffable aspects of human existence: reflections on birth and death, the origin, inception and divine purpose of life.

This may be demonstrated through the names given by various cultures throughout the ages, which speak of a living history of the religious or spiritual cosmologies of cultures (Smith, 1971):

- **Tholos** (Greek), which means rotunda, circular tomb with a beehive shape.
- **Omphalos** (Greek), which means umbilici, the centre, the central part.
- **Tegurium** (Latin), which means shrine, round roof covering an altar or a sarcophagus (often a pointed dome).
- **Domus** (Latin), which means house (Domus Dei: the House of God).
- **Kalubé** (Syrian & Palestinian), which is used for religious buildings and tabernacles.
- **Sacred baetyl** (Syrian & Palestinian), which means tabernacle, to manifest the divine.
- **Vihâra** (Sanskrit and Buddhist tradition), referring to spiritual community dwellings.
- **Dom** (German, Icelandic and Danish), which means cathedral.
- **Dome** (English), for Cathedral in the late 17^{th} Century (also town House, guild hall, and city meeting house).
- **Dominical** (French), which means “Day of God”.

Domes have also represented cosmological concepts such as:

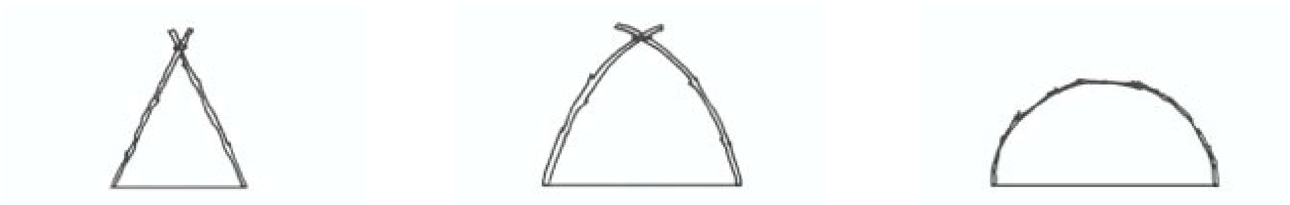
- **The Cosmic Egg**
  In Egypt, India, Persia and Greece, they are seen as representing the egg at the origin of creation.
- **The Celestial Helmet**
  This is related to the Cosmic Egg in Palestinian, Christian and Hebraic traditions, and represents the priest of heaven, or divine power and authority.
- **The lotus**
  In the Egyptian and Indian spiritual traditions, the lotus is seen as a manifestation of divinity.

1.2 EARLY HISTORY & TECHNICAL EVOLUTION

The earliest stages in the evolution of the vault and dome occurred prior to written records. Thus, while it is not possible to write about this evolution with historical certainty – particularly at a global scale and across a wide range of cultures – anthropologists, archaeologists and historians can help us to form a good interpretation. The following gives a brief outline of some of these technical evolutions in a very generalized manner; however, it is important to note that local factors particular to culture and context (e.g. individual cultural trends, economic stability, technical developments, climate, and availability of materials) are the most relevant factors influencing innovation in masonry.

Nomadic dwellings: It has been shown that the dome shape emerged in many different cultures concurrently, in the form of primitive hut architecture. In these nomadic states of human civilization, people's link with the cosmos through the starlit sky (e.g. for navigation) was a crucial means for survival. Dwellings were frequently symbolic of this link, taking on a circular form, covered with domes or conical forms constructed with branches (Smith, 1971).

It is feasible that conical huts evolved first, as they were simple, rapid to build and break down for transportation: a circular plan of branches anchored into the ground and joined together at the top (e.g. the North American Indian tipi) *(Fig. 1)*. In parallel, the branch dome evolved. Also based upon a circular plan, the branches were bent radially into the form of a dome and sometimes linked with rings of bundled branches for stability. These structures, which were often covered with stretched skin or thatch, can still be found in present day nomadic cultures (e.g. in the Horn of Africa). The conical form, which was fixed to the ground level, later evolved into a conical roof atop a cylindrical base. The result is the typical circular hut, which may still be found – in some cases, ubiquitously – in many developing countries *(Fig. 2)* (Smith, 1971).



Conical dome *Ovoid dome* *Semicircular dome*

Fig. 1 – Early development of branch domes

Fig. 2 – African bulbous (branches covered with skin/ canvas); Qubâb round huts (Syria, Somalia, Ethiopia, etc.)

Simple branch constructions later evolved into an organised and studied carpentry, stronger and more durable, which allowed for the construction of wider spans and higher rises. Wooden domes were at a time more prevalent in global architecture due their lightness, flexibility and in some cases earthquake resistance. However, being prone to fire and rot, wooden domes were less permanent than masonry.

As mentioned, innovation in arches, vaults and domes is also intimately linked with the constraints of place and climate. From Neolithic times, where earth, stone or other masonry materials were prevalent, crude early masonry structures and the knowledge of working with stone and earth also emerged. More durable than wooden structures, with heavy load bearing walls, masonry structures were preferred for their protection and relative permanence. For this reason, over time, masonry gained widespread usage for funereal monuments and religious buildings (Besenval, 1984) (Woolley & Mallowan, 1976).

Early proto-domes were developed in the monolithic masonry constructions of Neolithic villages and tombs. The earliest known examples, beehive-shaped corbelled domes of adobe block, were built in Khirokitia, Cyprus around 6000 BC. This corbeled dome, or *tholos*, was a common built form in Neolithic Mediterranean cultures. In many cases, the precursors to tholos structures were round-form dwellings with a central stone pillar, which were capped with megalithic stone slabs in radial formation. It is possible to imagine the eventual development from a radial flat slab roof technology, to a corbelled dome technology, such as can be seen in Etruscan and Mycenaean tombs *(Fig. 6)*. It is also possible to see in these examples a transition from corbelled arches to radial or "true" arches (although round arches can be found in other locations at earlier periods).

Once nomadic lifestyles gave way to more sedentary agrarian cultures, human settlement patterns evolved from the round hut to the square form, likely on account of increased density and peri-urban dwelling patterns. Some dwellings were still covered by (often hemispherical) domes, a form which is to this day still common in Islamic architecture (e.g. along the Mediterranean coast and in Egypt).

Even in ancient times, there was a great exchange of techniques from one part of the world to others through travel and trade. Masonry was the material of choice in ancient Mesopotamia, where ancient trade routes flourished throughout the lands of the Sumerians, Babylonians, Persians, and ancient Egyptians. Stone carvings of the Assyrians indicate strangely domical architecture (Paterson 1913); some of the earliest beehive corbelled structures can be found in present day Syria (Mecca, 2009). Further, the exchange of cultures, tools, techniques and craftsmen along the ancient trade routes of the Mediterranean (including Celtic, Etruscan, Phoenician) are well documented by historians (Jarzombek, 2009), (Houben & Guillaud, 1994), (Pankhurst, 1999). The bible documents in detail the masonry craftsmanship of Solomon's temple, built by Phoenicians, who are predominantly known as only traders in early Mediterranean culture (Edey, 1974).

The oldest vaults still standing were built with adobe (raw earth) around 1300 BC in Egypt, the vaults of the Ramasseum in the "rest" of Thebes, the granaries of Ramses II temple *(Fig. 6)*. Yet a negative consequence of its durability was that masonry structures lacked lightness, requiring heavy load bearing walls on account of their weight and thrust. They were consequently more labour intensive to build, more difficult to make stable, and more sensitive to earthquakes. In fact, a great deal of the technical development in masonry over the ages has been devoted to solving the challenge of its weight; making structures physically lighter, taller, better lit and ventilated, and more seismically resistant. For example, the Romans excelled in the construction of monolithic stone masonry, brick masonry and concrete. They had the technology to move massive materials at grand scales and full mastery of the round arch. Yet, when it came to vault and dome construction, the Romans invested considerable effort in reducing the weight of masonry, through the use of coffering and pozzolanic concretes with mixed materials of diminishing density. The Pantheon, built 75-138 AD in Rome, serves as an excellent example of the height of Roman innovation (∅ 43.30 m, pozzolanic concrete and brick) *(Fig. 6)* (Lancaster, 2005).

In the Byzantine period, early basilicas were developed with increasing complexity in form, lightness, and earthquake resistance. *Barrel vaults* were intersected to generate the form of groin vaults. The basic square house with a round dome became more technically advanced with the introduction of the dome on pendentives, which carried loads more efficiently to the foundations. The dome on pendentives likewise evolved into more sophisticated forms, until the 6th century construction of the basilica of Hagia Sophia in Constantinople *(Fig. 6)*. Lunettes were built into the base of the structurally daring hemispherical dome on pendentives (Ø 34-35 m, 55.6 m high), to counter the thrust of the dome. The Hagia Sophia represents the greatest development of Byzantine architecture, and it was later taken as one of the most influential pieces of architecture for both the West and the East (Mainstone, 1965).

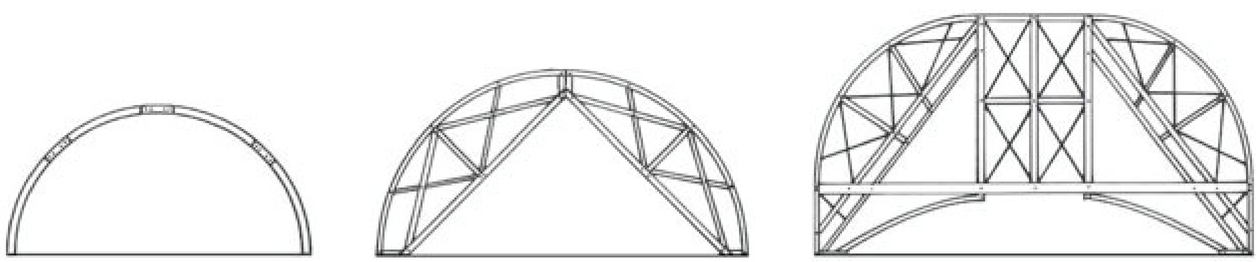
Early Arab science advanced significantly with the translation of Euclid's Elements into Arabic (c. 800 AD), predating its presence in the rest of Europe by more than 300 years (Necipoğlu, 1995). This sparked a great renaissance in Islamic mathematics and geometry, which led to the introduction of double and multiple pointed arches of many geometries, as well as the calculation of more elaborate structures, shapes and ornamentation. At the time, this knowledge was transcribed into practical manuals for masons, which were used to translate information between mathematicians and craftsmen (Necipoğlu, 1995).

The result was a flourishing of craftsmanship, advancements in vault forms, complicated intersecting vault geometries, masonry bond patterns, ornamentation and acoustics in Islamic, Timurid and Turkmen architecture.

The Ottoman period inherited the great traditions of Islamic architecture as well as the architecture of the Byzantines. The great Ottoman architect Sinan built mosques throughout the extent of the Ottoman empire, which further elaborated upon the forms and methods of the Hagia Sophia.

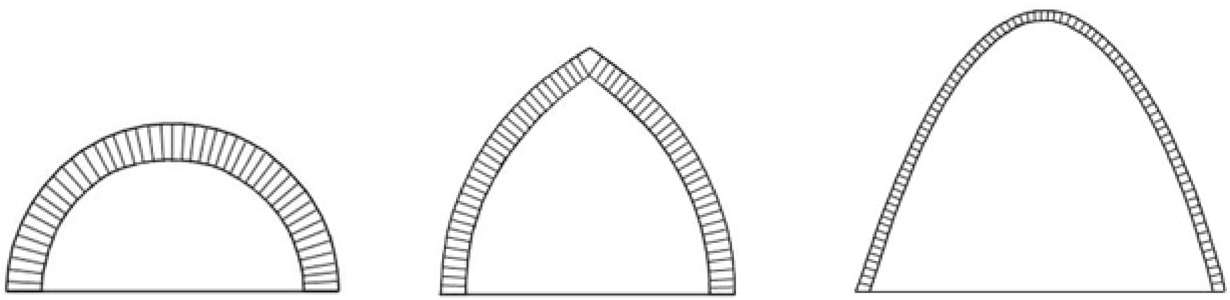
There came the age of the great European cathedral architecture (Huerta, 2012), (Evans, 1995), (Fitchen, 1961), (Mackenzie, 1821), (Porter, 1982), (Willis, 1835), (Cram, 2002), (Wilson, 1990). Master masonry guilds developed, which passed down knowledge through numerous generations. This period is associated with another series of great developments in complexity, as the symbolic forms of early Christiandom (e.g. the cross as the nave, transept and crossing of the cathedral) developed from simpler space partitions to more elaborate forms.

The constructive techniques developed by Gothic cathedral masons required a great knowledge and facility of wooden formwork construction *(Fig. 3)*. It is asserted by some scholars that the development of Gothic vaulted architecture proceeded from exchanges with the technological advance of wood craftsmen from ship construction (Rabasa Díaz, 2009). In any case, the evolution of wooden formwork construction necessarily coincided with the evolution of larger span, more complicated and more structurally efficient masonry structures *(Fig. 4)*.



Short span | *Medium span* | *Large spans*

Fig. 3 – Technical development of wooden formwork



Hemispherical vault | *Pointed vault* | *Catenary vault*

Fig. 4 – Technical development of masonry

These vaults and domes were not developed only for technical reasons, but also represented the spiritual pursuit to build spaces reflecting sublime religious experience. In this period of time, religious fervour and generous financing from the church and wealthy patrons allowed the generations of master craftsmen to slowly transform the heavy and massive Romanesque style into structures of lightness with the flamboyance of high Gothic architecture (Victor Hugo, 1831).

Groin vaults evolved into ribbed vaults, with ribs present at the intersections of vaults. Ribbed vault complexities developed with the great advances in stone "stereotomy" (or stone cutting) techniques (Evans, 1995). After crusaders returned to Europe with knowledge of the pointed arch from the Islamic world, romanesque single-point geometries evolved into Gothic "ogival" (or pointed) arches, vaults and domes. Master masons became very resourceful about where to best position their loads.

Thus, geometries became more structurally efficient; thicknesses could thereby be reduced to the limit of the crushing strength of the materials and great portions of the facades could be opened for interior lighting. Taller, lighter, more slender and more daring cathedrals were built: higher vaults, wider spans, more daring thickness/span and slenderness ratios. Structures could climb to previously unachieved heights as the complexity of buttress systems and carefully positioned loads (e.g. pinnacles or loads above ogival vaulting) allowed for the careful control of thrust forces. Fan vaulting flaunted the rules of compression forms by transferring loads through surfaces as thin as 10 cm (Leedy, 1980).

These developments were often the result of technical experiments, which were not always immediately successful. However, at this point in time, the collective knowledge gathered gave master masons very good tools for estimating the proportions of thickness (Huerta, 2012). Failed attempts did of course occur – particularly in cases in which insufficient consideration was given to the foundations. However, these proportion estimates were relatively effective in establishing stable structures. Additionally, since the small stresses in masonry allows structures to be virtually scalable, the use of scaled models operated as an effective tool to test elaborate constructions at smaller scales, while gaining skill and income (Leedy, 1980).

After the height of this period, structures began to evolve towards an economy of materials and cost. The Duomo of the Cathedral di Santa Maria del Fiore was designed and built by Brunelleschi in Florence, 1421-35. The Duomo had an elaborate octagonal facetted dome with a double shell (∅ 45.4 m), which was designed to be built of fired brick without the aid of expensive formwork or scaffolding systems *(Fig. 6)* (Mainstone, 1977).

Yet thickness ratios were still based upon regular geometries, and not structural forms. The thickness required of hemispherical arches was reduced by the use of equilateral (two point) arches and ogival pointed arches of the gothics. However, it would not be until after the age of the cathedral mason that a better knowledge of structural form would be developed (Huerta, 2008). With the introduction of Robert Hooke's 2nd Law in 1675 and improved understanding of the forces in arches, vaults and domes, methods were developed for the analysis of built structures. Poleni first applied modern structural theory in 1748 with the principle of the hanging chain to evaluation the extensive cracking in the dome St. Peter's of Rome (∅ 41.47 m) *(See Section 2.2: Catenary Method, p. 25)* (Cowan, 1977). Later, this efficient "catenary" geometry was first implemented for the design of new structures by Sir Christopher Wren, in his design of St. Paul's Cathedral, London (∅ 31 m) *(Fig. 6)* (Cowan, 1977) (Addis, 2000).

Nevertheless, the great age of master masonry had already come to a close, with many factors converging to quicken its disappearance – in particular, the advent of industrialization and the profusion of industrial materials such as iron, steel and later reinforced concrete. The entry of industrialized building materials into the mainstream market, which could boast cheaper and faster construction costs and reduced labour, quickly overtook traditional materials and construction methods.

The dominance of such materials in the mainstream building market coincided with major advances in modern engineering: the development of "Elasticity Theory". This paradigm of structural engineering based upon stress and a uniform and predictable modulus of elasticity of a material, which not appropriate for the analysis of masonry structures, quickened the lost knowledge in the calculation of the masonry arch, vault and dome.

Architecture and structural engineering diverged further into disparate specialized disciplines, and the synthetic knowledge of master builders was lost. The industrialization of the West, along with capitalism and market economy of supply and demand, targeted increasing productivity and decreasing cost. This had the effect of inflation and increasing cost of labour; thus labour-intensive construction systems such as masonry suffered most.

Some great masters emerged very late in this industrialized time, including Antonio Gaudí in Spain and Nari Gandhi in India. Gaudí's extravagant craftsmanship combined with a sophisticated use of structural forms generated by networks of hanging chains *(See Section 2.2: Catenary Method, p.25)*.

Later, there were certain short periods of great revival in the masonry arts, when an efficient and cost-effective technology could be deployed with sufficient knowledge and skill. For instance, more than a thousand buildings with vaults and domes were built by the R. Guastavino Company on the east coast of the United States in the years between the late 1889 and 1962 (Ochsendorf, 2010), (Collins, 1968).

The thin-tile vaulting technology, a traditional Catalan vaulting technique, which was further developed by the Guastavino's, served as a cost-effective means for roof systems for buildings which at the time had few competitive alternatives. Many of the greatest innovators in vaulted masonry in the last century have specialized in thin, shell structures, including Pier Luiji Nervi, Eladio Dieste, Eduardo Torroja, Félix Candela, and Heinz Isler.

In the present day, a limited number of vernacular vault building traditions still exist around the world. In general, formwork-less construction methods (which do not require formwork and thus reduce the cost of construction), have become the prevailing forms of vaulted masonry. This includes Nubian vaulting, Catalan or thin-tile vaulting, Mexican vaulting, and “Free-spanning” Vaulting which was developed at the Auroville Earth Institute.

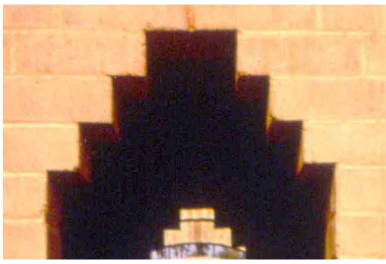
1.3 DIVERSITY – PAST & PRESENT

A nearly infinite variety of arch types, styles and shapes can be found all over the world. Influenced mainly by the creativity and traditions of individual cultures, this variety has also evolved alongside the constraints of given localities throughout history, for instance, the climate, the type of locally available materials, access to technologies and presence of technical know-how. Vaults and domes may also be found in a great diversity of shapes, though they tend to be less stylistically diverse than arches.

Semicircular arch – France

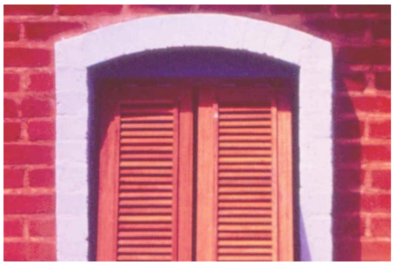
Bucket arch – Brazil

Segmental arch – Auroville

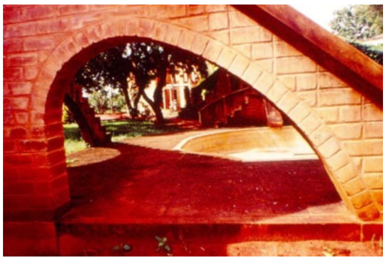


Corbelled arch – Auroville

Corbelled arch – Greece, 1325 BC



Segmental arch – Ivory Coast



Rampant arch – Auroville

Pointed segmental arch - Somalia

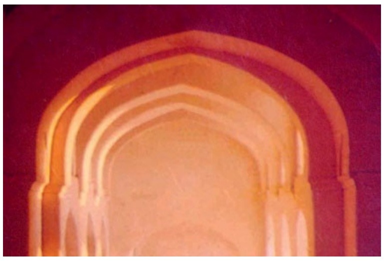
Pointed arch – Gujarat, India

Pointed "slab" arch – India

Pointed "slab" arch – India

Pointed arch - Turkey

Bucket pointed arch – Turkey



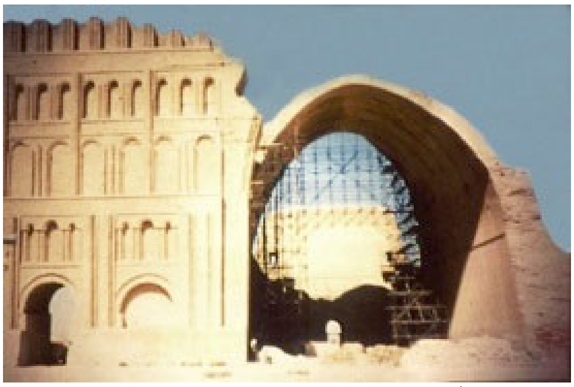
Bucket pointed arch – Rajasthan

Bucket pointed arch – Agra, India

Fig. 5 – Various arches around the world

Ramasseum – Gourna, Egypt, ca. 1300 BC
Nubian vaults ~ 4 m span, adobe

Tomb of Atreus – Mycenae, Greece, 1325 BC
Pointed dome ~ ∅18 m, stone



Iwan of the Palace of Ctesiphon – Iraq, ca. 6th century
Vault 21 m span, 30 m high, fired brick

Imedghassen Mausoleum – Algeria, ca. 4th century BC
Corbelled dome ~ ∅58.86 m, stone

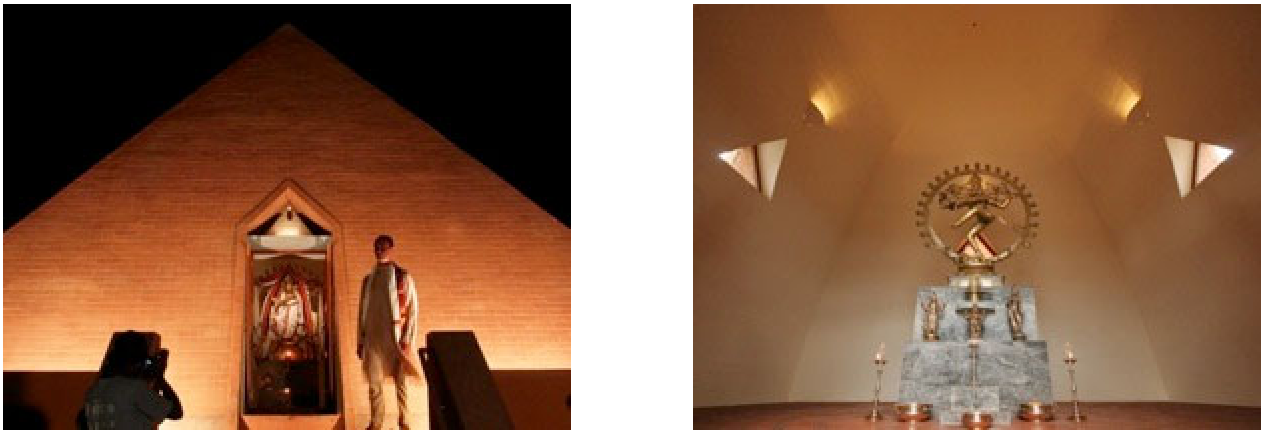
The Pantheon – Rome, Italy, 75-138 AD
Dome ∅43.30 m, pozzolanic concrete and brick

Basilica Hagia Sophia - Istanbul, Turkey, 532-537 AD
Hemispherical dome on pendentives 34 x 35 m

Santa Maria del Fiore – Florence, Italy, 1421-35 AD
Facetted dome ∅45.4 m, fired brick

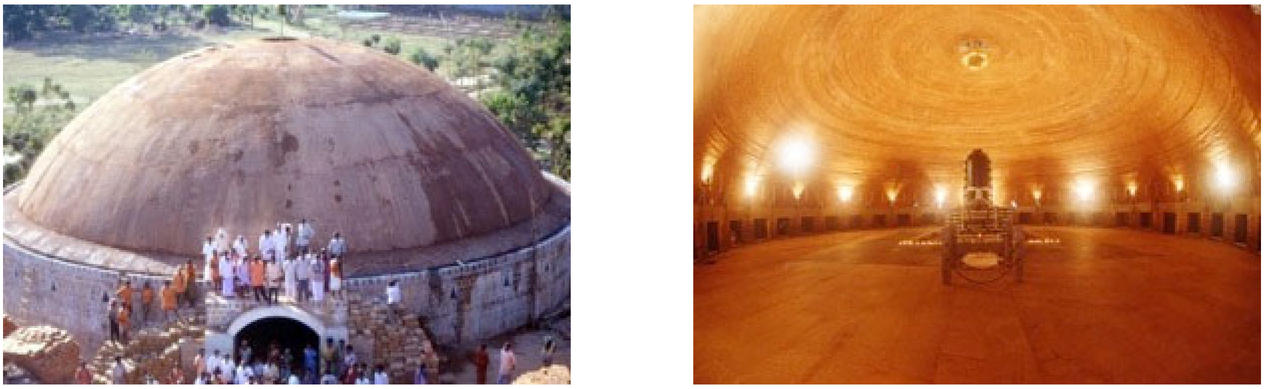
St. Paul's Cathedral – London, England, 1710 AD
Dome ∅31 m, stone

Fig. 6 – Various vaults and domes around the world

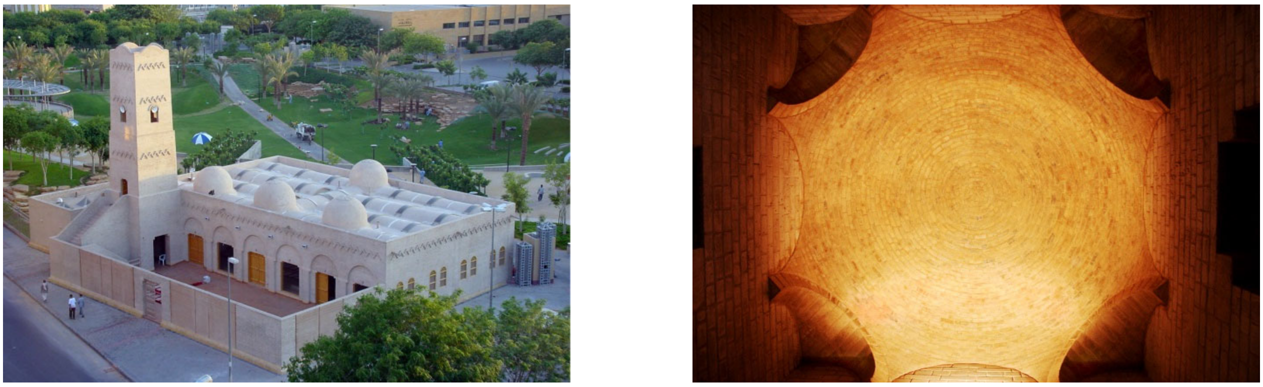


Karneshwar Nataraja Temple – Auroville, India, 2006. Pyramid with Pointed dome 6.0 m span, 5.30 m rise, CSEB

Ermitage – Chinna Kalapet, India, 2006. Equilateral vault 4 m span, CSEB



Dhyanalinga Temple – Poondi, India, 1998. Segmental elliptical ∅22.16 m, fired brick



Al Medy Mosque – Riyadh, Saudi Arabia, 2004. Semicircular domes ∅3.01 m, CSEB

Fig. 7 – Selected work of the Auroville Earth Institute

1.4 GEOMETRY, TYPOLOGY & TERMINOLOGY

There are many different geometric shapes for arches. This includes single-point geometries (such as semicircular or segmental arches), multiple-point geometries such as pointed or bucket arches and other forms such as catenary arches.

In all cases, the geometric relationships are as follows:

- An arch is most typically generated by the rotation of radii from one or more points.
- A vault is most typically generated by the extrusion of an arch section into space.
- A dome is generated by the rotation of an arch section about a vertical axis. Domes can be built on circular or quadrangular plans.

The main exceptions to these principles are as follows:

- A dome on squinches is generated by a succession of increasing arches, starting from the corners. As long as the generating arch is not pointed, this squinch resembles half a cone or a portion of a cone.
- A faceted dome is generated by the intersection of vaults, similarly to the cloister dome. However, it is built on a faceted plan rather than a square or quadrangular plan, as in the case of the cloister dome.
- A groin vault (or groin dome) is generated by the intersection of two vaults (or domes) crossing each other. Typically, this crossing is perpendicular.
- A cloister dome is also generated by the intersection of two vaults crossing each other, as in the case of the groin vault. The groin of the groin vault becomes the principle geometry, between which facetted surfaces are drawn for a cloister dome. The arch section of the cloister dome is not evident from the side of the dome, but exists only within its cross sections.

Fig. 8 outlines only the basic typologies, which can be commonly found. The variety of arches, vaults and domes is much greater than these simplified categories. Note that the name of the arch is defined by its intrados geometry.

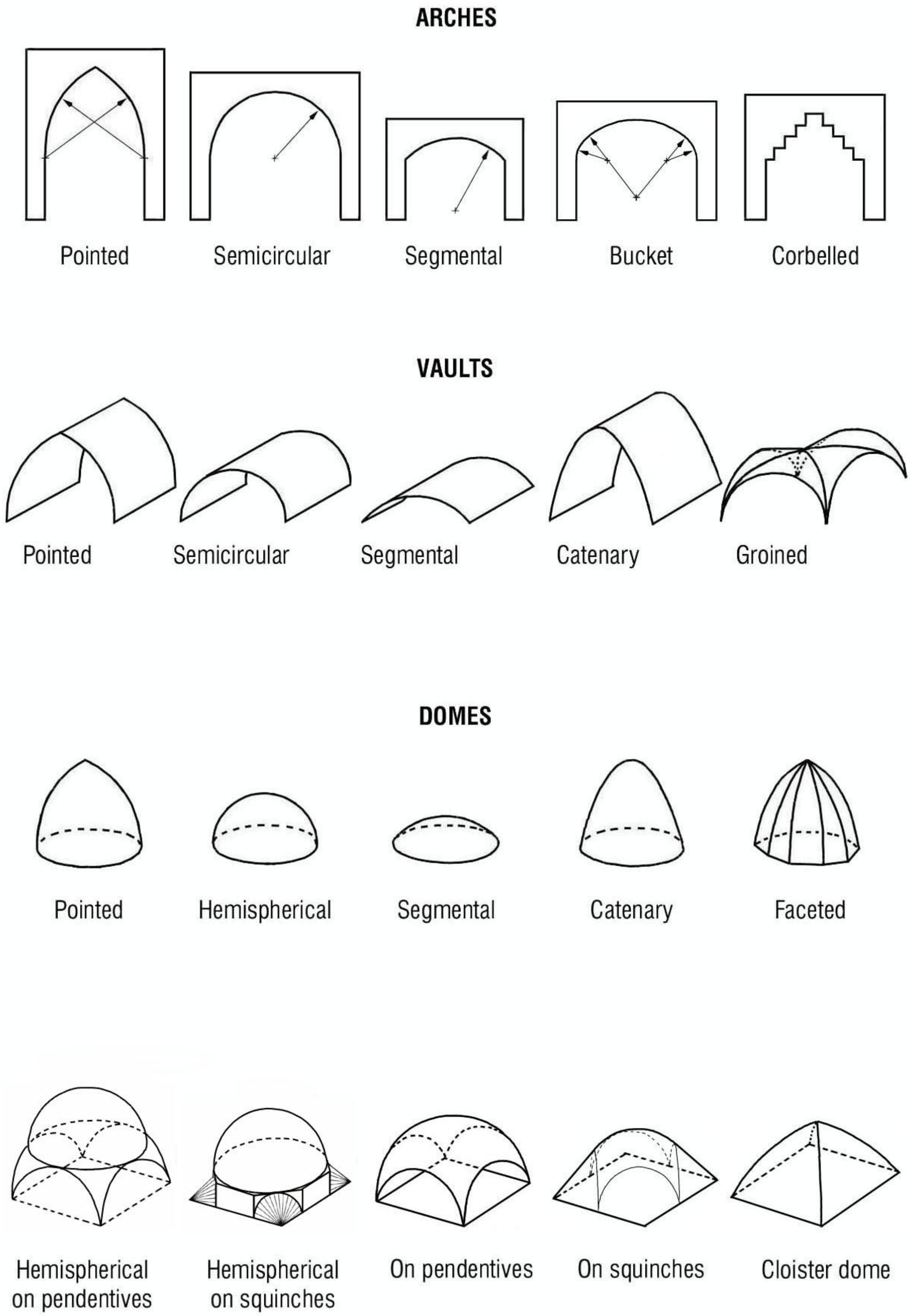


Fig. 8 – Typologies of AVD

The following diagram demonstrates the basic key terms which apply to arches, vaults, domes and the centrings which may be used for their construction. This can be used as a basic visual reference for beginners. All terms are outlined in greater detail in the glossary at the back of the manual.

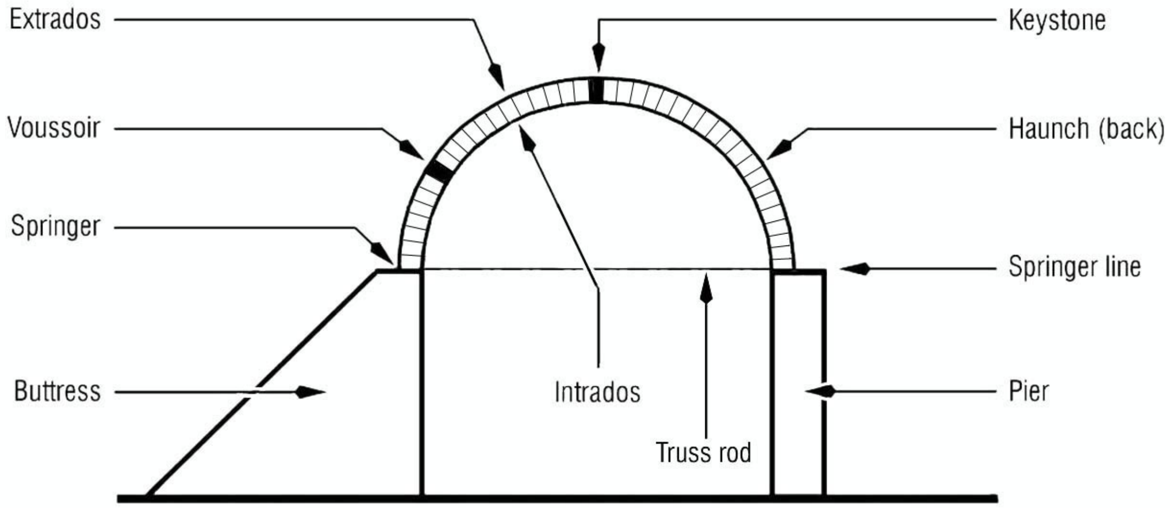


Fig. 9 – Terminology for an arch

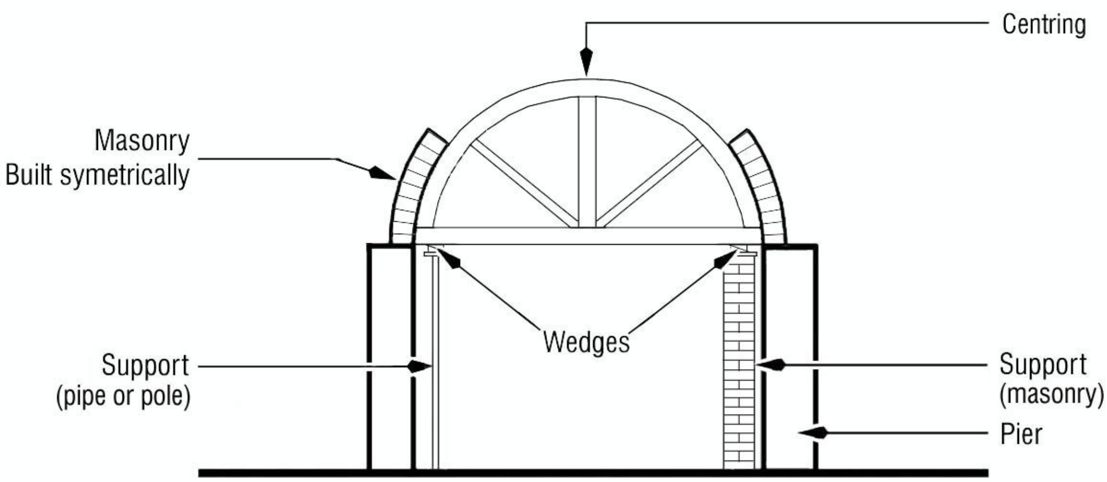
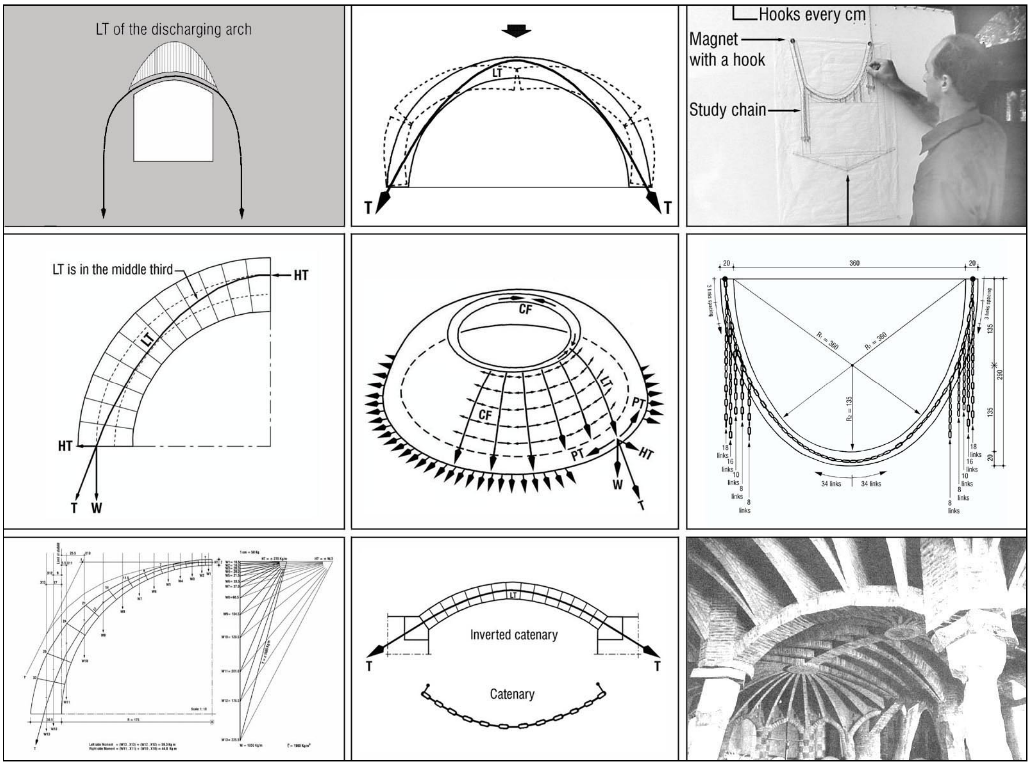


Fig. 10 – Terminology for a centring

Note that both supports of a centring should be of the same material

2. STABILITY OF ARCHES & VAULTS



2.1 BASIC STRUCTURAL PRINCIPLES

2.1.1 About Masonry

Characteristically, masonry is a heterogeneous material (Huerta, 2001). That means that masonry is not a single homogenous material with uniform properties throughout, like steel, but rather a composite of many smaller units (i.e. voussoirs or bricks), bound together by a mortar in an infinite number of possible configurations (i.e. with a bond pattern). Masonry is naturally very strong in compression and very weak in tension.

Jacques Heyman, the father of Plasticity Theory, laid out the following three assumptions of masonry in the Limit Analysis Framework of masonry (Heyman, 1966): Analysis of masonry should consider that masonry has 1. No tensile strength; 2. Infinite compressive strength (stresses are low enough so that crushing does not occur); 3. Sliding does not occur (enough friction between voussoirs). The same assumptions apply for earthen masonry.

Unreinforced masonry structures have very low stress levels. Therefore, masonry rarely fails because of material failure (e.g. crushing). Stability, not strength, governs the safety of masonry. And stability (or equilibrium) is based upon *geometrical* constraints.

2.1.2 Forces Acting in Arches & Vaults

Note: For the sake of simplicity, only the term "arch" is used in this section; however, since a vault is analysed in cross section – as an arch – this approach is also valid for single-curvature vaulting.

The force acting in arches is a compressive vector force known as compressive *Thrust (T)*. Vectors have a magnitude and a direction, and may be evaluated by their vertical and horizontal vector components or as a vector resultant. Thrust is the vector resultant of the 1. Weight of the masonry and 2. Horizontal thrust (or outward pushing force) as the weight of the masonry is transferred through the geometry of the arch.

The vertical component of thrust is determined by the *Weight of the Masonry (W)*. For analysis, the arch is typically cut up into equiangular voussoirs, and the self-weight of each voussoir is calculated. This may include only the *Dead Load* of the masonry, or both Dead Load and *Live Load*.

The horizontal component of thrust, or *Horizontal Thrust (HT)*, is determined by the geometry of the arch. The shallower an arch is, the greater the value the horizontal thrust will be. Therefore, the horizontal thrust can be minimized by the optimisation of the arch profile. This horizontal thrust value remains consistent throughout any given arch; it acts upon both springers and throughout the masonry. When analysing half of an arch, HT is indicated at the top of the arch to represent the equilibrium of the second half of the arch *(Fig. 11)*.

As weight is transferred down through the arch, the thrust pushes downwards and outwards with a trajectory, which depends upon the weight and the geometric profile of the arch. This successive pushing action from one voussoir of the arch to the next may be represented by a theoretical *Line of Thrust (LT)*. Technically, LT is the locus of the intersection of internal resultant forces acting in the arch. It is the trajectory and position of this Line of Thrust, which determines the stability of the masonry arch.

Limit analysis states that "*the vault will stand as long as a thrust line can be found that fits within its section*".

T = Thrust
(Resultant force of the weight and horizontal thrust)

W = Weight of the masonry and overload (DL and LL)
(Vertical force component)

HT = Horizontal Thrust of the masonry
(Horizontal force component)

LT = Line of Thrust
(Represents the successive action of the voussoirs)

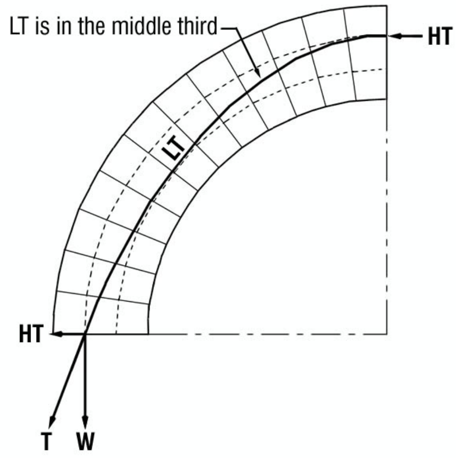


Fig. 11 – Forces in arches and vaults

Thrust values can be minimized by the optimisation of the arch profile (i.e. by adjusting self-weight and/or the geometry of the arch), however, there will always be a thrust in arches, vaults and domes, which must be balanced by means of buttresses, tension ties or ring beams *(See Section 2.7: Equilibration of Thrust, p. 70)*.

2.1.3 Funicular & Catenary Geometries

The Line of Thrust is in all cases a curve with a *funicular* geometry, a curve of efficient, natural load transfer in masonry. *Funicular geometries* are geometries, in which self-weight loads are carried in pure axial tension or compression (which eliminate bending moments). Arches can have various shapes and sizes, but the Line of Thrust of an arch is always a funicular geometry, which transmits forces in axial compression.

A funicular geometry is always unique to a particular loading condition. The Line of Thrust of an evenly loaded arch (e.g. an arch of equal thickness) always follows the shape of an inverted *catenary* curve. A *Catenary* is the curve assumed by a freely suspended chain subjected only to its self-weight and gravitational forces, which hangs in pure axial tension. The centre line of the links represents a funicular line of pure tensile stress, under the condition of <u>equal loading along the curve</u>. Mathematically speaking, this curve is the graph of a hyperbolic cosine function.

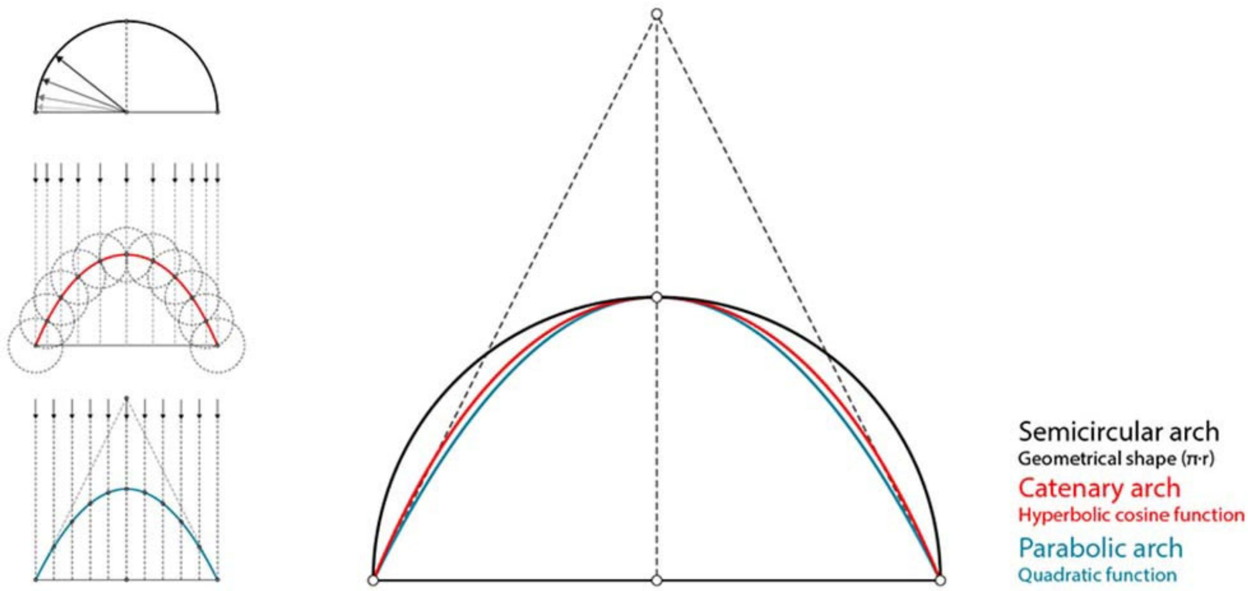


Fig. 12 – Geometric differences between a semicircle, parabola and catenary curve

Robert Hooke first demonstrated in 1675, "*As hangs the flexible line, so but inverted will stand the rigid arch*." In other words: the catenary geometry of a hanging chain (which hangs in axial tension), when flipped upside-down, describes the geometry of an ideal arch (which stands in axial compression). The catenary or hanging chain is a pure tension form; the inverted catenary is a pure compression form.

Therefore, in any given arch, the Line of Thrust is a funicular curve of pure compressive stress; and in an arch of equal thickness, the Line of Thrust is a catenary curve of pure compressive stress.

In an arch with the geometry of a perfect inverted catenary curve, the voussoirs can be visualised as corresponding to the links of the chain. Just as the links of the chain are in tension, the voussoirs of the inverted catenary arch are in compression and the Line of Thrust (LT) is centred in the voussoirs.

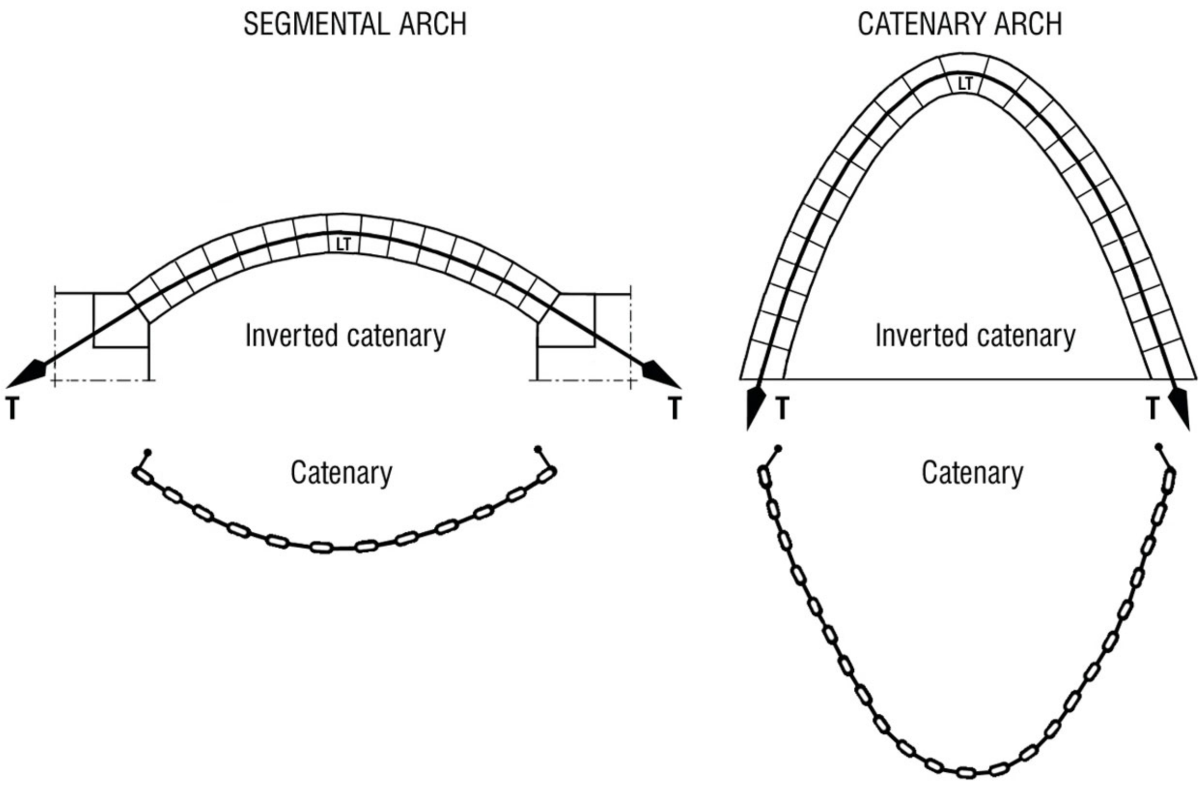


Fig. 13 – Catenary curves and arches

A catenary arch can vary infinitely, assuming either a deep or a shallow geometry. A deep catenary exerts very little horizontal thrust, and a shallow catenary exerts a lot of horizontal thrust. (Imagine how much force is required to pull a hanging chain until it is perfectly straight.)

The Line of Thrust is centred in the arch <u>only</u> in the case of catenary arches. In all other types of arches of equal thickness, the LT cannot be centred within the arch. In some cases, the LT of other arch geometries can come <u>near to</u> the centre line. For example, the LT in segmental arches comes close to the centre of the arch. As a catenary curve becomes more horizontal, it comes closer to the geometry of a segmental arch. As it becomes more vertical, it departs more and more from the geometry of a single-point or semicircular arch.

If one hangs an extra weight on a hanging chain, it develops a "kink", like a necklace with a pendentive. This kinked chain is no longer catenary, but a funicular geometry for the condition of the added weight. Similarly, when considered in the arch, this added weight would represent an extra load placed over the masonry arch (e.g. a larger keystone of a pointed arch).

2.1.4 Principles of Stability

This may be formulated as the most basic rule of the stability of arches: **<u>As long as the Line of Thrust travels through the cross-section of the masonry, the arch stays in compression and is stable</u>.**

This is the foundational principle of the *Safe Theorem* (or lower-bound theorem), developed within the framework of Limit Analysis for masonry structures by Heyman (Heyman, 1966). The safe theorem proves that an unreinforced masonry arch will stand, even if only one possible solution can be found within the arch.

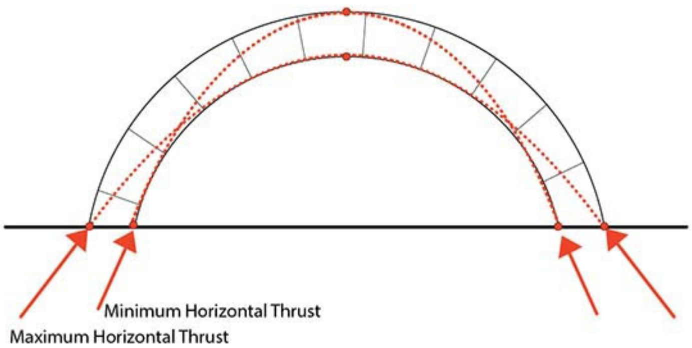


Fig. 14 – Range of position of Line of Thrust

There is an infinite range of possible lines of thrust which can travel within an arch, within which there are two limits: the most horizontal Line of Thrust (i.e. with the maximum horizontal thrust) and the most vertical Line of Thrust (i.e. with the minimum horizontal thrust). Within this range, moreover, it is not necessary or possible to know the exact state of the internal forces in an arch. The Line of Thrust can change position in response to any change of loading, *abutment* or even environmental conditions, but its exact position cannot be precisely known.

Once the Line of Thrust touches either the *intrados* (interior face) or the *extrados* (exterior face) of an arch, a crack will form on the opposite face, creating a hinge or point of rotation. Compression forces cannot be transmitted through a crack, so it is necessary for the Line of Thrust to pass through the hinge point. According, the exact position of a Line of Thrust is only known (or determinate) in the case in the case of the 3-hinge arch, as LT is fixed through each hinge point).

When the Line of Thrust touches the intrados of the arch, the arch will form a hinge point on the intrados and the arch will tend to burst outwards. Similarly, when the Line of Thrust touches the extrados of the arch, the arch will form a hinge point on the extrados and the arch will tend to rotate and collapse inwards.

However, cracks are normally safe and typically occur in arches. As long as no more than three hinges form in the arch, the arch is stable. If <u>more than three hinges</u> form in the arch (typically 4 or 5), a failure mechanism can form which will cause the arch to collapse. If a load is applied and increased so that the Line of Thrust becomes tangent to the arch in four or more positions, the arch will collapse.

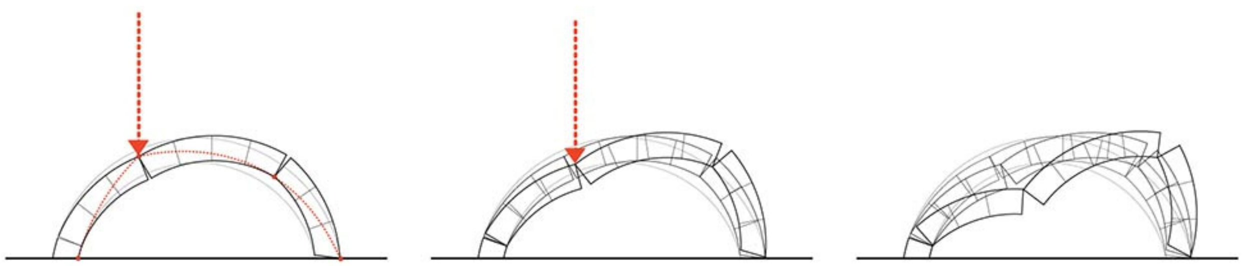


Fig. 15 – Critical load causing collapse (after Heyman, 1995)

In order to prevent the possibility of the LT exiting the arch, or the development of 4 hinges, a geometric safety factor can be considered for arches.

This may be formulated as another rule of the stability of arches: **An arch or vault is stable and safe as long as the Line of Thrust remains within the middle third of the arch section.**

This is a condition of safe stability, as defined by the “Middle third rule” of the Limit Analysis framework of masonry structures (Heyman, 1988). Disregard for this rule may cause deformations, which can lead to collapse.

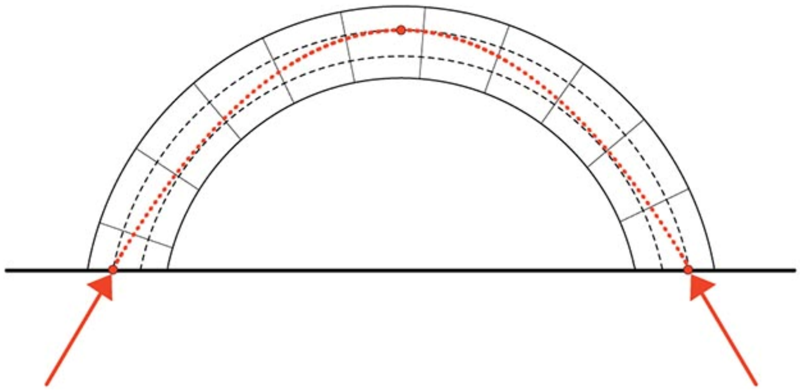


Fig. 16 – The middle third as a geometrical safety factor

2.1.5 Examples: Influence of the Arch Thickness on Stability

Accordingly, it is self-evident that the thickness of an arch influences its stability. We have seen that the Line of Thrust assumes the shape of an inverted catenary curve and should always remain in the middle third of the arch. However, we have also seen that there is an infinite variety of arch geometries within which this catenary LT must past. Let us look at two examples of arch typologies, the semicircular arch and the Egyptian arch, to understand the relationship between thickness, span-thickness ratios, and safe positioning of the LT.

2.1.5.1 Example 1: Semicircular Arch

Semicircular arches have a very different profile compared to the catenary curve. Therefore, LT typically deviates significantly from the centre line, and dramatically changes the compressive stress distribution in the masonry. In order to achieve a LT in the middle third of the arch, the thickness can be considered in relation to the span. This ratio can be used to ensure an appropriate proportional thickness. This was the strategy that enabled master European cathedral masons to accurately assess stable structures without having knowledge of the internal forces in arches, vaults and domes (Huerta, 2012).

As a classical “rule of thumb”, semicircular arches should have a minimum thickness of:

$t \geq \frac{S}{5}$ (Where t is the thickness and S the span)

This demonstrates very clearly that a free-standing semicircular arch (e.g. without any overload or load on the haunches) needs to be very thick to be stable: A 6 m span arch requires a 1.20 m thickness to establish LT within the limits of the middle third.

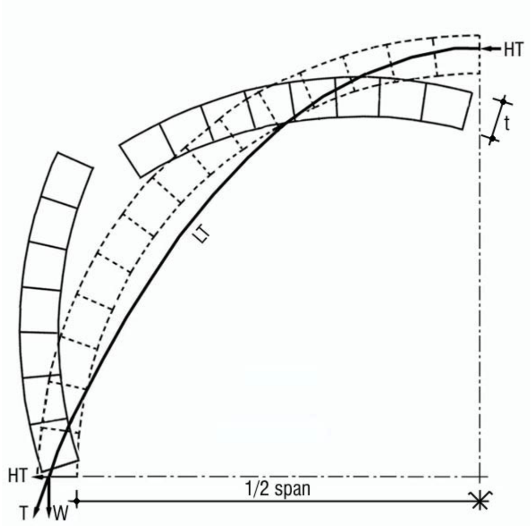


Fig. 17 – Instability in an arch which is too thin:
t = S/20

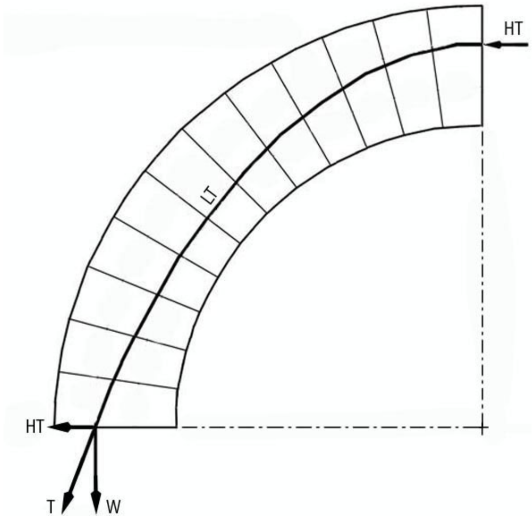


Fig. 18 – Stability with the proper arch thickness:
t = S/5

2.1.5.2 Example 2: Egyptian Arch

Similarly, an Egyptian arch needs to be relatively thick to be stable. The "rule of thumb" for Egyptian arches is that:

$t \geq \frac{S}{7}$ (Where *t* is the thickness and *S* the span)

Therefore, an Egyptian arch of 5 m span will require a 71.5 cm thickness, in order to establish LT at the inner limit of the middle third.

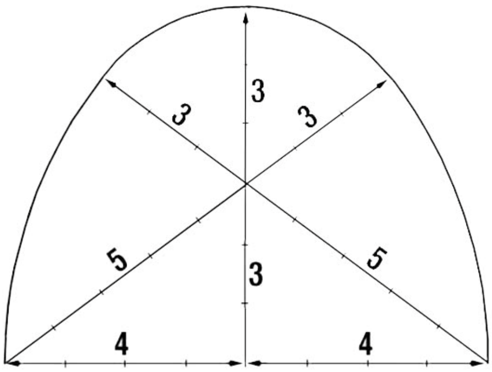


Fig. 19 – Proportions of the Egyptian arch,
based on the Pythagorean triangle 3,4,5

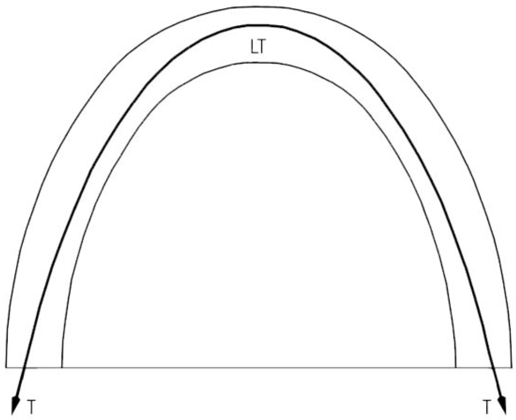


Fig. 20 – Stability of the Egyptian arch with
the proper thickness: t = S/7

To increase the stability of an arch, one should try to achieve a LT as close as possible to the centre line of that arch.

2.1.6 Examples: Strategies for Arch Stabilisation

2.1.6.1 Example 1: Loading & Overloading

A heavy central load is applied at the crown of the arch, or the shape is disproportional.

The LT becomes tangent to the intrados and extrados surface of the arch in more than 3 positions, which will cause failure.

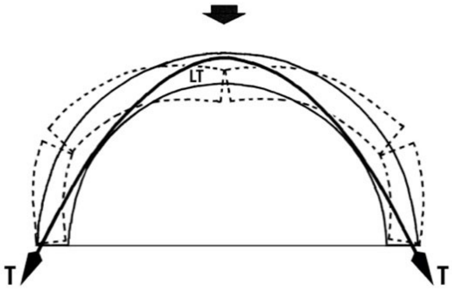


Fig. 21 – Central load and failure

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Remedies

Change the shape of the arch

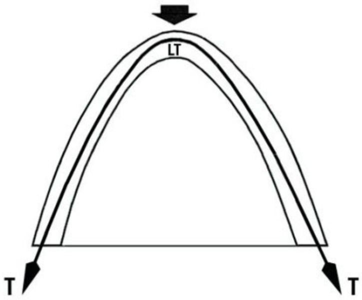

Fig. 22 – Catenary arch

Maintain shape/thickness; load the haunches

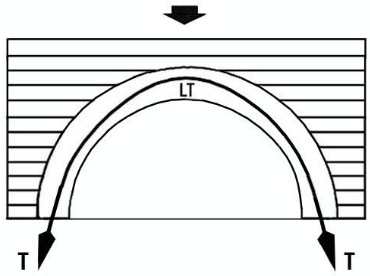

Fig. 23 – Loaded arch

2.1.6.2 Example 2: Symmetrical & Asymmetrical Loading

Depending on the load applied to the arch, the LT will assume a particular curve and the arch can be shaped accordingly: Here both symmetrical and asymmetrical loads are applied.

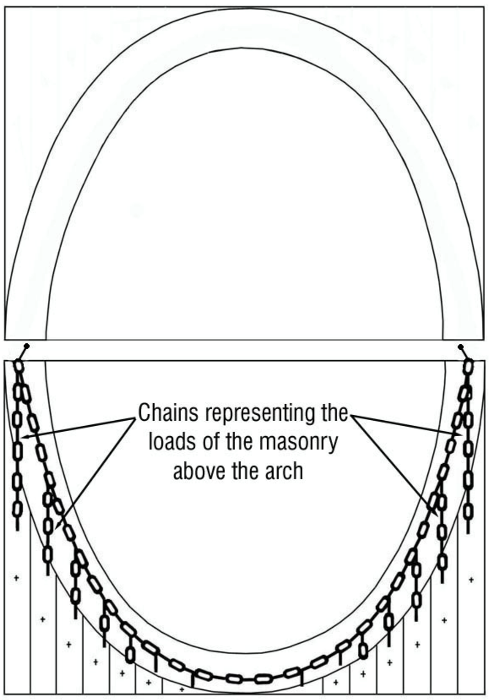

Fig. 24 – Symmetrical load and funicular curve

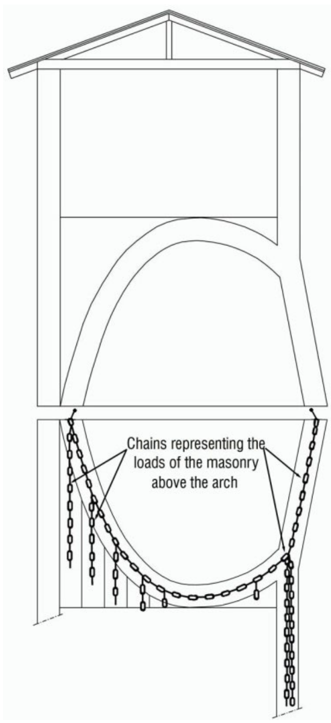

Fig. 25 – Asymmetrical load and funicular curve

2.1.7 Boundary Conditions: LT in Walls, Piers & Foundations

The Line of Thrust of an unreinforced masonry structure must not only pass within the middle third of the arch itself, but it must also remain within the middle third of the supporting piers and foundations, in order for the load to be transmitted to the ground. The mass of the masonry itself may be used to direct the path of the load transfer.

2.1.7.1 Example 1: Effect of Overloading on LT

Adding loads above the arch will modify the Line of Thrust in the masonry.

As seen previously, "free standing" semicircular barrel vaults need to be very thick, on account of their geometry and required span/ thickness ratio. Yet the thickness of a semicircular arch can be reduced if the haunches are loaded. Loading the haunches of this arch will have three effects:

1. LT will be drawn into the middle third of the arch and the arch will become stable.
2. The load on the haunches will load the pier and kink the thrust into a more vertical position. Thus, the width of the pier can also be reduced.
3. The horizontal thrust will not be affected, while the weight and the magnitude of the resultant thrust will be increased.

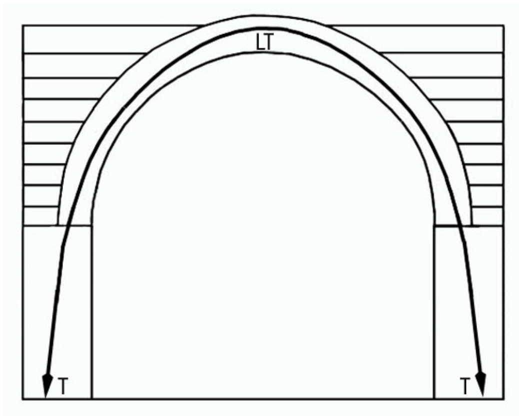

Fig. 26 – Reduced thickness with load on the haunches: t = S/10

2.1.7.2 Example 2: LT in Piers and Foundations

In this example, the LT is in the middle third of a "free standing" arch, but <u>not</u> in the middle third of the pier. As the entire structure is unreinforced, the pier is not wide enough: The LT exits the pier above the foundation, causing the structure to collapse.

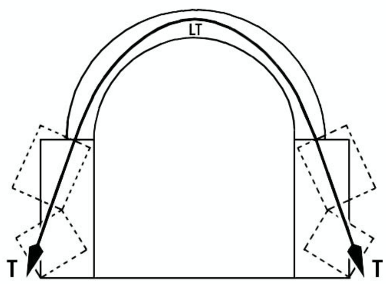

Fig. 27 – Pier too thin and failure

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Remedies:

Increase the width of the pier or, if it is a vault, add regularly spaced buttresses

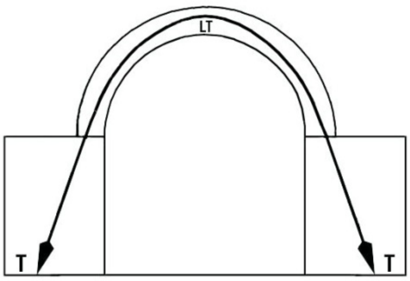


Fig. 28 – Wider pier

Load the haunches of the arch to change the angle of the thrust

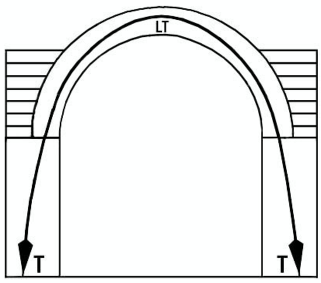


Fig. 29 – Load on the haunches

2.1.7.3 Example 3: LT of an Arch Within a Wall

When an arch is heavily overloaded by the weight of a wall, the position of LT changes in response to the load. In some cases, it no longer passes within the arch but through the masonry above (seen as the higher of the two LT's in *Fig. 30*).

Thus, the principal Line of Thrust behaves like a discharging arch transferring the majority of the load over the arch. A secondary Line of Thrust will still run somewhere within the arch itself; however, this secondary LT will support only the self-weight of the arch and the wall section between the arch and the principle LT. This parabolic shaped section of the wall, which slightly increases the magnitude of the secondary LT, will cause this LT to pass closer to the extrados of the arch and to exit nearer the intrados.

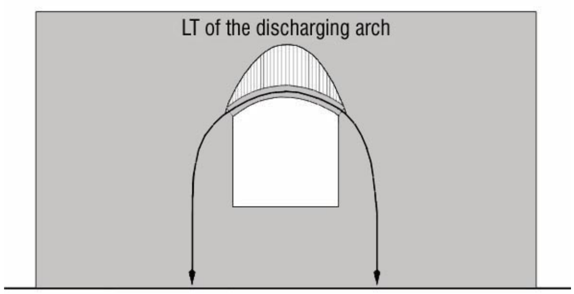


Fig. 30 – Modification of LT in a wall

2.1.8 Influence of Mortar on Stability

The various methods for calculating the stability of arches and vaults (*See Sections* 2.2, 2.3 and 2.4), do not consider the effect of the mortar on the strength of vaulted structures. Calculations assume compression-only structures which are built with dry stacked masonry.

Generally speaking, mortar binds blocks together and transmits compression forces. At the intrados of an arch, forces are transferred directly from block to block: they touch each other. At the extrados of an arch, the contact is ensured by the mortar, which transmits compression forces when it is dry.

In general, mortars have a relatively low tensile strength. However, this can considerably increase the loadbearing capacity of an arch. Nevertheless, the tensile capacity of masonry is unreliable and therefore should not be considered for the stability of vaulted structures.

When vaults and domes are built with the Nubian or Free Spanning techniques, the quality of the mortar is essential to stick the blocks onto each other *(See Section 4.4.3: Vaults and Domes Built with the Nubian Technique, p. 91)*. This cohesion is only required while building the structure. Once the structure is completed, the mortar achieves its dry compressive strength, is consolidated by compression forces in the arch, and the transmission of the forces occurs through the mortar.

2.2 CATENARY METHOD

2.2.1 Background & Aim

As early as 1675, an English engineer of the 16^{th} century, Robert Hooke, made the correlation between the tensile stress in a chain and the compressive stress in an arch. Hooke wrote a Latin anagram in the margins of an obscure publication: "*As hangs the flexible line, so but inverted will stand the rigid arch*" (1675); meaning that the geometry of a hanging chain, which describes a pure tension form, when flipped upside-down, describes a pure compression form for an arch. This provocative inscription later became known as Hooke's 2^{nd} Law.

It wasn't until 1743, however, that the principles described by Hooke were first applied by Poleni for the stability analysis of the dome of St. Peter's. Poleni used a hanging chain to demonstrate that the line of thrust remains safely within the cross-section of the structure, and that it was safe despite the concern of significant cracking. Sir Christopher Wren was the first to apply Hooke's principles in the design and construction of buildings, among them St. Paul's Cathedral, London (with a catenary relieving dome) and The Wren Library, Cambridge (with catenary inverted arch foundations).

In the late 19^{th} and early 20^{th} centuries, knowledge of graphical analysis spread throughout Europe. Antonio Gaudí, Spanish architect of the early 20^{th} century, extensively developed and deployed the catenary method. He designed structures, such as the Colonia Güell, with complex networks of hanging chains. He studied the loads applied to arches, piers and columns with masterful precision, maintaining funicular geometries throughout the entire system of masonry. The piers or columns supporting an arch were often given the inclination of the line of thrust.

The catenary method has been developed substantially by the Auroville Earth Institute into its present form. Since the catenary method is not extremely accurate and cannot determine the magnitudes of forces in the system, it is no longer in use as an actual form-finding method. Nevertheless, its pedagogical function is indispensable.

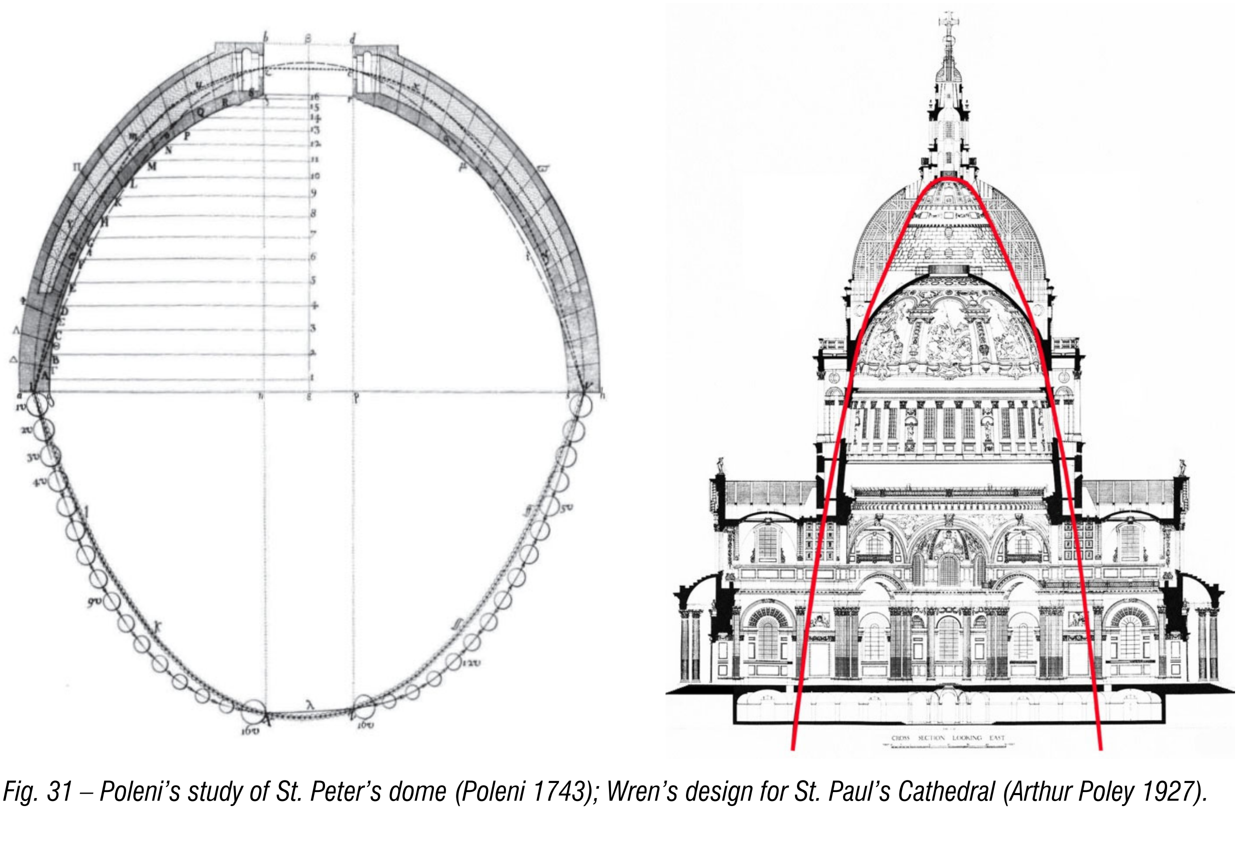


Fig. 31 – Poleni's study of St. Peter's dome (Poleni 1743); Wren's design for St. Paul's Cathedral (Arthur Poley 1927).

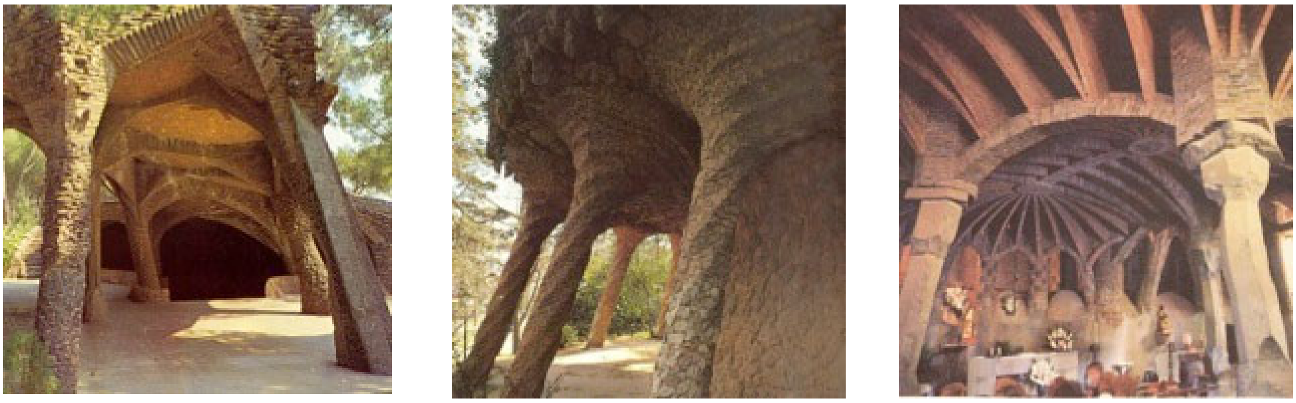


Fig. 32 – Works of Gaudí in Barcelona, Spain, with funicular arches and columns

2.2.2 Principle

The method described here is for the analysis of a vault. However, the word “arch” will be used for simplicity, as it represents the vault section. A board is required to conduct the catenary study. The section of the desired arch is drawn, upside-down, at a scale fitting the study board. A chain with the length of the arch centre line is hung freely on the board. It will assume the curve of a catenary.

The chain is then loaded with other small chains, which represent the various loads required to bring the line of thrust within the middle third of the arch. This modified curve will no longer be catenary, but *funicular*, representing the most efficient curve for the case of the applied loads. It represents the line of thrust.

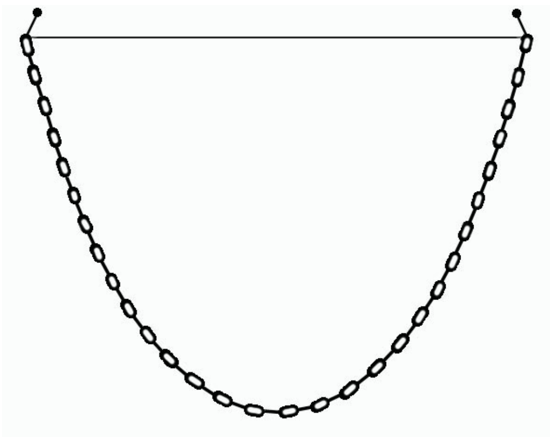


Fig. 33 – Catenary assumed by a chain hung freely on the board

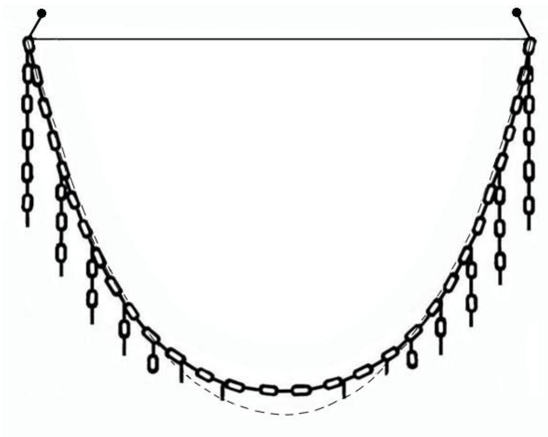


Fig. 34 – Funicular curve assumed by a chain loaded with various chains

The catenary method indicates only the exact and ideal curve of the line of thrust, which represents the line of compressive stress in the arch. However, it does not give the magnitude of these forces. The funicular method is required to determine the value of the forces acting in the arch *(See Section 2.3: Funicular Method, p. 30)*.

The catenary method requires some basic equipment, which is outlined in the following list. The number of small chains and the number of links required may vary according to the size and type of arch studied.

Equipment required for the catenary method

- A white study board of about 1 m square. It should be equipped with hooks at the top on which to hang the chain (with a spacing of 1 cm).
- It should also be possible to fix the chains anywhere on the study board.

  (With a plywood board, the chain can be pinned onto the board; With a metal sheet, magnetic hooks can be anchored to the board.)
- A chain with a length of 1 to 2 m, with links of about 1 cm in length.
- Many small chains with various numbers of links which can be used to load the principle chain:

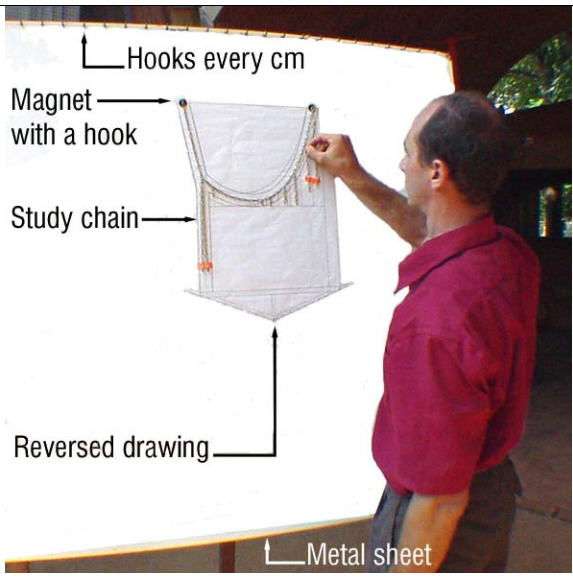


Fig. 35 – Studying an arch on the study board

- 20 Nos. with 1 link	- 16 Nos. with 6 links	- 12 Nos. with 12 links
- 20 Nos. with 2 links	- 16 Nos. with 7 links	- 10 Nos. with 14 links
- 20 Nos. with 3 links	- 12 Nos. with 8 links	- 10 Nos. with 16 links
- 16 Nos. with 4 links	- 12 Nos. with 9 links	- 10 Nos. with 18 links
- 16 Nos. with 5 links	- 12 Nos. with 10 links	- 8 Nos. with 20 links

2.2.3 Method

The aim of the method is to load the chain with small loads (small chains) in order to reposition the principle chain so that it lies within the middle third of the arch section. The method presented here assumes that the arch/vault is free standing. It can have any shape, thickness or span, but is studied alone as though it were for a roof.

1. Draw the section of the desired arch using a scale (1/5, 1/10 or 1/20) which fits on the catenary study board. Note that the bigger the drawing is, the better it is for the accuracy of the study. Draw also on this section the middle third of the selected arch.

2. Calculate the length (in m) of the centre line of the arch:

   $$A_{Centerline} = \frac{\pi(R + 1/2t)\alpha}{180}$$ Where: R = Radius (m), t = thickness (m), α = angle of the arch

   Or measure it to scale if the arch is too complicated.
   Note that the centre line of the arch does not correspond with the line of thrust. Nevertheless, this approximation is sufficient for the study.

3. Reverse the drawing on the study board and hang the chain with the length of the arch centre line (at the chosen scale).

   Unless the desired arch has a pure catenary section, the chain will rarely be within the middle third of the arch.

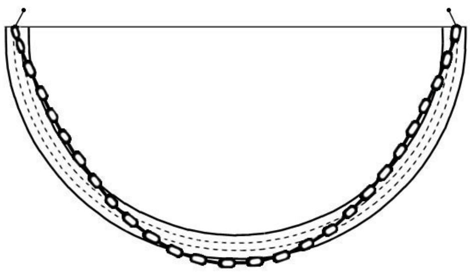


Fig. 36 – Chain hung freely on the study board

4. Load the chain symmetrically with weights: the small chains with the pre-determined number of links. The spacing between these loads should be regular, between 2 and 5 links depending on the arch size and shape.

   Note that the weights near the springer have very little influence on the curve at this portion of the arch, however, they influence the magnitude of the thrust and the whole shape of LT.

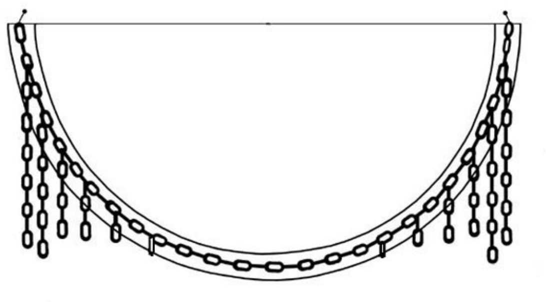


Fig. 37 – Chain loaded with small chains

5. Adjust the loads until the principle chain remains within the middle third of the arch. Note that the main chain, representing the line of thrust, should follow these thumb rules:

Arch type	Entry of HT	Exit of LT
➢ Segmental arches	Centre of the arch	Centre of the arch
➢ Bucket arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Semicircular arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Egyptian arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Catenary arches	Centre of the arch	Centre of the arch
➢ Equilateral arches	Touches the intrados of the arch	At 2/3 from the arch intrados
➢ Pointed arches	In the intrados third of the arch	At 2/3 from the arch intrados
➢ Corbelled arches	Centre of the arch	Centre of the arch

6. After applying the loads, the length of the chain may need to be slightly adjusted: either a little longer or shorter in order for it to remain in the middle third. Once this has been done, count the number of links of the chain for the entire arch.

7. The length of segments is the number of links corresponding to the spacing between the loads applied on the chain.

   Note that the centre of these segments corresponds to the location of the loads. The length of the top segments will be different, as they need to be adjusted according to the length of the arch.

8. Record the various loads hooked onto the chain and note the projected floor spacing between them.

9. Reference the segments with the loads applied.

   These segments are counted from top to bottom of the actual arch for both right and left sides.

   Count how many links each segment has: the number of links corresponding to the spacing of the loads plus the number of links of the small loading chains (if any).

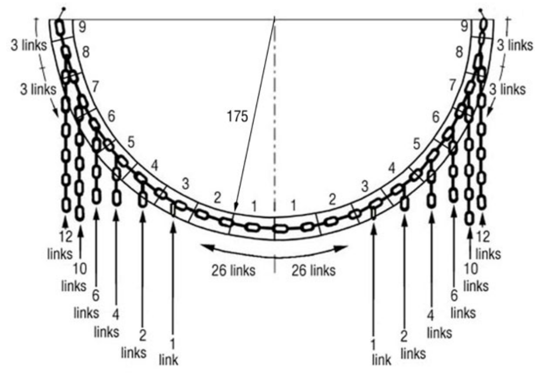


Fig. 38 – Segments with the number of links

10. Calculate the length (L, in m) of a segment at the scale of the arch:

$$L = \frac{(\text{No. of links per segment}) \times (\text{Length of chain}) \times (\text{Drawing scale})}{\text{No. of Links}}$$

Where:

- Length of chain = Exact length (m) of the chain on the catenary model
(the length of the arch centre line if it has not been adjusted, or the new length of chain after adjustment)

- Links Nos. = Total No. of links for the entire chain on the catenary model.

Example with 3 links per segment: $L = \frac{(3 links \times 0.58 m \times 10)}{52 links} = 0.334 m$

11. Calculate the linear weight of all segments (kg/m):

$$W_{Segment} = L \times t \times p \times \frac{(\text{Total No. of links per segment})}{(\text{No. of links per segment})}$$

Where: L = Segment length (m), t = thickness (m), ρ = volumic mass (kg/m^3)
Total Nos. of links per segment = Number of links of the segment (including the load if any)
Nos. of links per segment = Number of links of the segment

12. The stability of the arch is now defined with theoretical loads applied to it with chains. The drawing of the arch can now be reversed and these loads have to be materialised by masonry.

13. The masonry required to load the arch can be represented either with vertical columns of masonry or with a smooth curve with increasing thickness of the arch from the bottom to top.
Measure on the drawing, or calculate if possible, the width between the vertical centre lines of the loads. Convert this dimension into m, according to the scale of the drawing.
Calculate the height (m) of the masonry which corresponds to the weight of this load:

$$Height = \frac{W_{Load}}{(\text{Width}_{\text{Load centre lines}}) \times \rho}$$

Where: ρ is the volumic mass (kg/m^3)
Width $_{\text{Load centre lines}}$ is in m

Note that the depth of the strip of arch to be analyzed has not yet been considered. Weights and loads are calculated per running metre.

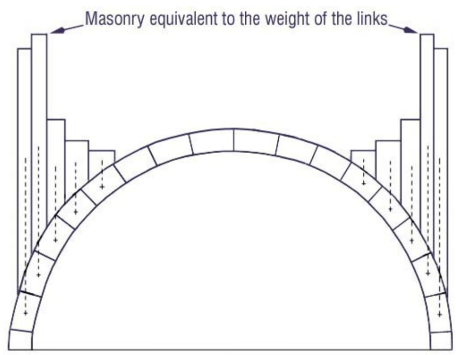


Fig. 39 – Represent theoretical weights of masonry

14. The masonry required on the haunches is rarely aesthetically harmonious. It is better to represent the loads with varying thicknesses from the bottom to top. The study can continue with the optimisation method *(See Section 2.4: Optimisation Method, p. 37)*.

2.3 FUNICULAR METHOD

2.3.1 Background & Aim

The development of funicular methods for the analysis of arches, vaults and domes has a considerably long history, which has been well documented in the writing of Santiago Huerta (Huerta, 2008). A very simplified account of this history follows.

Graphical analysis emerged from empirical methods in the manipulation of hanging chain models. We have seen already that the catenary method can show the exact position of the line of thrust. However, it does not give any information about the magnitude of this thrust.

Several mid-19th Century engineers, in Germany, France and England, simultaneously developed funicular methods to calculate the forces acting in an arch (Gertsner, 1831), (Mery, 1840), and (Moseley, 1835). (Huerta, 2008)
It was understood that external loads applied to a hanging chain or string could be summarized by a diagram later called a *Force diagram*. This force diagram, a vector diagram at a particular scale, summarized the equilibrium of the *Form diagram* or the cross section of the arch being studied. The relationship of form and force diagram is a simple rule of parallelity: the external loads from the form diagram are drawn parallel in the force diagram. Once a scale is given for this diagram, it is possible to determine the magnitude of internal forces at work in the arch. This allows the catenary method to provide critical information about loading.

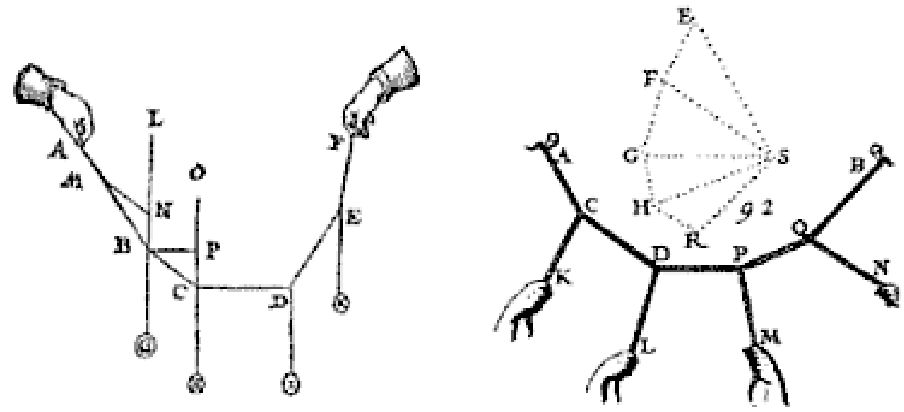


Fig. 40 – Analogue hanging chain studies with a force diagram (Stevin 1586 & Varignon 1725)

In the 1966, these principles were codified and proven by the British structural masonry engineer Jacques Heyman, who developed the Limit Analysis Framework of masonry (or "Plasticity Theory"). This reverses about 200 years' worth of thinking in the field of structural masonry engineering, and proves that the methods of medieval master builders were in fact correct.

These graphical principles have been used by the Auroville Earth Institute over the course of the last 25 years. The special distinctions of the funicular methods developed by the Earth Institute include that:

- Provisional horizontal thrust values are determined by proportional entry and exit positions at the springing and crown of the vault. These provisional figures have been determined by extensive empirical study of many different arch geometries over the years of the Earth Institute's existence (e.g. statistical probability of thrust line position for semicircular, segmental, pointed, bucket, etc. arch types).
- Additionally, the AVEI Optimisation method has been developed *(See Section 2.4: Optimisation Method, p. 37)* for the structural design of vaults an optimal section of varying thickness. Such vaults are designed for construction without centring with the AVEI "Free Spanning" technique *(See Section 4.3: "Free Spanning" Technique, p. 87)*.

2.3.2 Principle

The funicular method employs a form diagram *(left)* which represents the cross section of the arch or vault to be studied, and a force diagram *(right)* which is a scaled diagram representing the internal forces of the arch. Since the resultant forces in the form and force diagram are parallel to one another, it is possible to transfer them from one diagram to the other by simply pulling them parallel. First a provisional funicular diagram is drawn. This provisional diagram is then used to determine the final forces in the arch by geometrical proof.

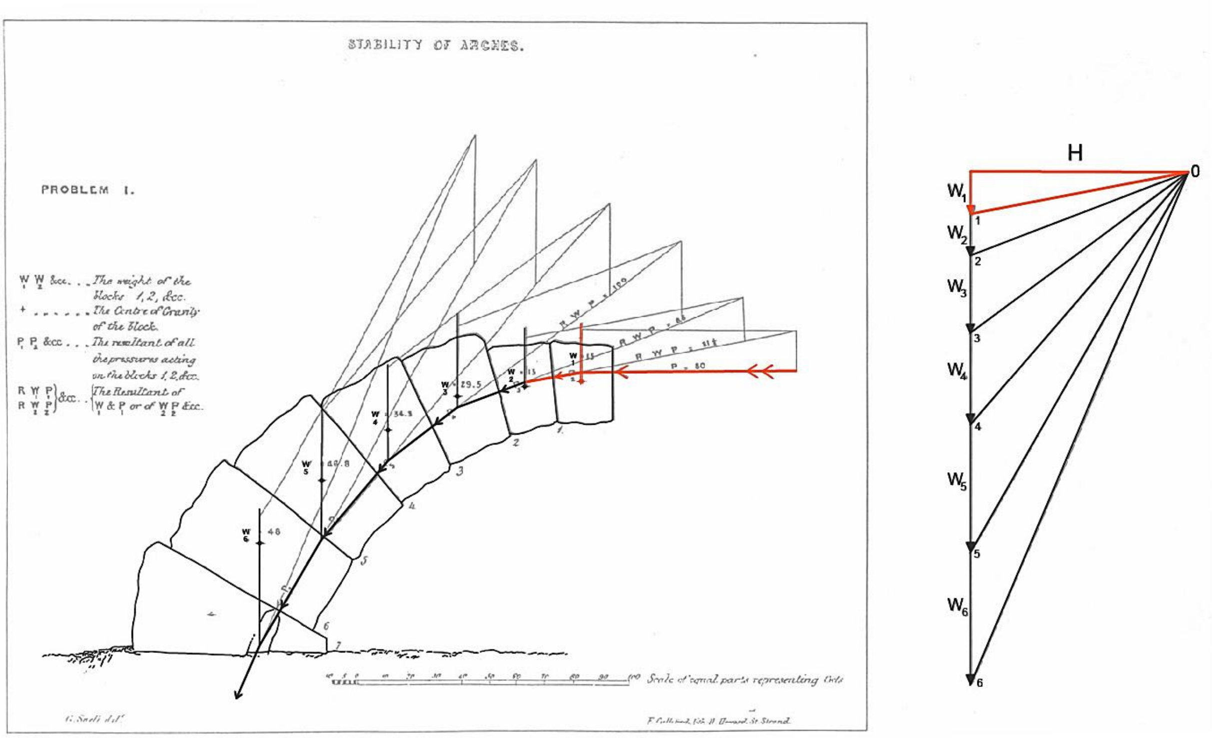


Fig. 41 – Form and force diagrams (after Snell 1846 & Huerta 2006)

The funicular method outlined in this document will assume the convention of analysing one half of the arch. This means that one half of the arch will be "cut" for analysis. HT is applied on top of the section of the arch as a reaction force representing the balance of the second half of the arch where it is cut.

Procedurally, half of the arch (the form diagram) is drawn at scale and divided into short segments, preferably of equal size. The segment weights are calculated and the centre of gravity (CG) of each segment is defined. Vertical working lines, where the weights are applied, are drawn through the CGs. A provisional force diagram is then drawn, beginning with the only known variable: the weight of voussoirs. A provisional horizontal thrust, HT', is selected to begin the study.

When HT encounters the working line of the CG of the first segment, the direction of LT will change. The Line of Thrust will become "kinked" at the position where each load is applied (along the working line of each centroid). The resultant force will likewise encounter the next segment's working line and will again change direction. This is repeated until LT encounters the last working line of the last CG. This final thrust resultant, T', will be used to determine the actual thrust, T, along with the direction and magnitude of the other force vectors in the arch.

This method, described in detail as follows, can be used to determine the magnitude of the thrust, to define where a LT passes in an arch, and therefore to determine whether the arch is stable and safe.

2.3.3 Method

1. Draw one half of the arch at the largest possible scale according to the span and shape of the arch (i.e. 1/2 or 1/5 or 1/10 or 1/20). Divide it into short segments, preferably of equal length.

2. Calculate the self-weight (kg) of all segments. The segment areas can be approximated as trapezoids. Note that this analysis of a semicircular segment as a trapezoid is sufficiently accurate for this study *(See Annex: Geometric Formulas, Centre of Gravity of a Segment, p. 111)*.

$$Weight_{segment} = m_e \times t \times d \times \rho$$

Where: m_e = median of the trapezoid (m), t = thickness (m), ρ = volumic mass (kg/m^3)
d = depth of the arch (m) – If the study is for a vault, the depth is taken as 1 metre.

For the purpose of this study, consider:

ρ CSEB = 1,900 kg/m^3
d = 1.0 m

3. Define the centre of gravity (CG) of each segment:

Draw the arch centreline.
The CG will be centred on the median of the segment.

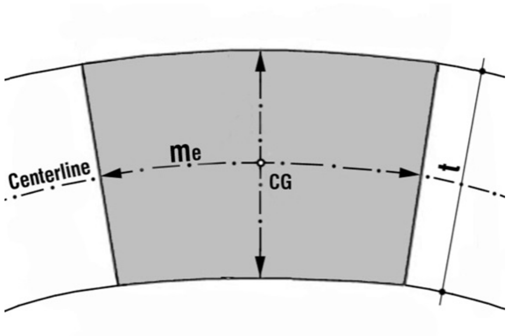


Fig. 42 – Centre of gravity of a segment

4. Draw vertical working lines from each CG and reference all segments for clarity in the drawing.

Note that the segments are referenced with a naming convention from top to bottom.

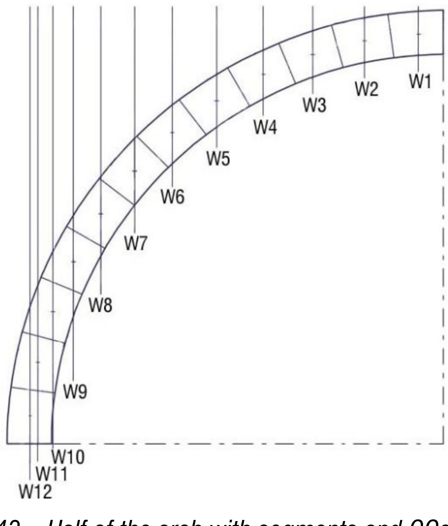


Fig. 43 – Half of the arch with segments and CGs

5. Add the weights of all segments to determine the total weight, W (in kg), of one half the arch.

6. Evaluate the provisional horizontal thrust, HT'. HT' is a provisional determination of the horizontal thrust value, which allows a trial funicular polygon to be drawn.

According to the shape, HT’ can be selected according to the following (to ensure proportional diagrams):

- Segmental arches : HT’ = ~ W
- Bucket arches : HT’ = ~ W
- Semicircular arches : HT’ = ~ W/2
- Egyptian arches : HT’ = ~ W/2
- Catenary arches : HT’ = ~ W/2
- Equilateral arches : HT’ = ~ W/2
- Pointed arches : HT’ = ~ W/2 or W/3
- Corbelled arches : HT’ = ~ W/3

7. Define a scale for the funicular diagram, according to the size and weight of the arch. Define the largest possible scale fitting the drawing, i.e.:
   1 cm = 10 kg
   1 cm = 20 kg
   1 cm = 50 kg

8. Begin to draw the trial funicular diagram:

   - Report all weights onto the vertical axis of the diagram. This is called the *load line*, and represents all external loads applied (the weight of each arch segment). If the loads are vertically applied (self-weight, acting with gravity), the load line will always be vertical.

   - Report HT’ on the diagram.

   - Draw the resultant forces by connecting the tip of HT’, called the *pole point*, to each weight on the load-line.

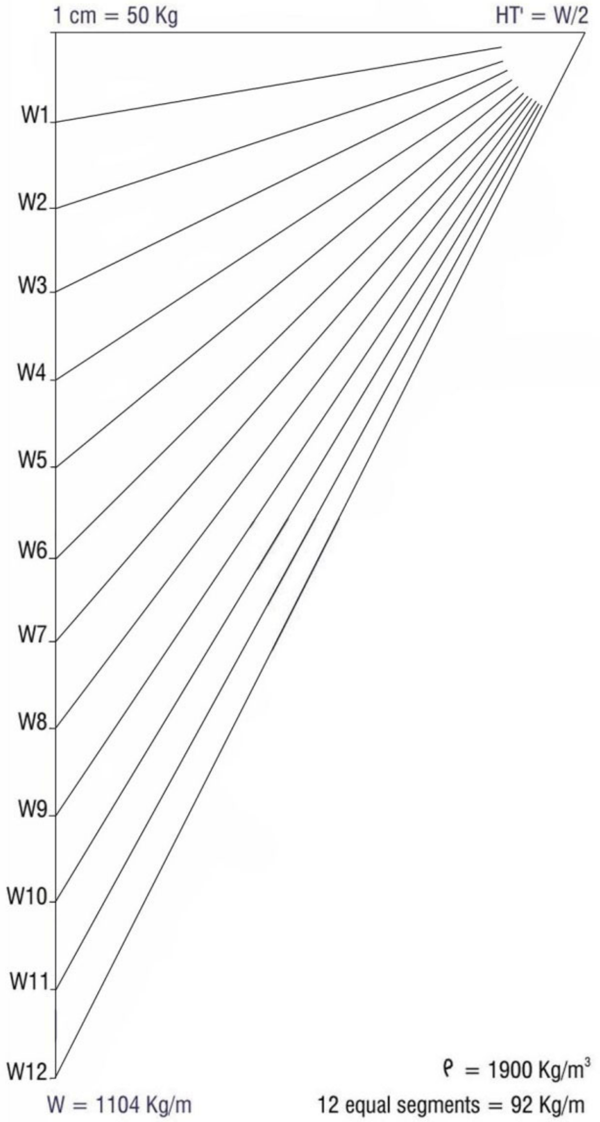


Fig. 44 – Funicular diagram with W & HT’

9. Transfer the resultant forces of the diagram, one after the other, onto the section of half the arch:
As these resultant forces are parallel in both diagrams, they can be transferred by pulling them parallel from the force diagram to the form diagram.

- Draw the line of HT’ and let it enter the arch according to this pattern for the typical arches:

Arch type	Entry of HT’	Exit of LT’
➢ Segmental arches	Centre of the arch	Centre of the arch
➢ Bucket arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Semicircular arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Egyptian arches	At 2/3 from the arch intrados	At 2/3 from the arch intrados
➢ Catenary arches	Centre of the arch	Centre of the arch
➢ Equilateral arches	Touches the intrados of the arch	At 2/3 from the arch intrados
➢ Pointed arches	In the intrados third of the arch	At 2/3 from the arch intrados
➢ Corbelled arches	Centre of the arch	Centre of the arch

- HT' remains horizontal until it encounters the vertical working line of the first segment.

- Draw the first resultant of the thrust, T_1' from the first vertical line:
  Transfer T_1' (the resultant of HT' and W_1) from the diagram. The LT will become "kinked" at the position where each load is applied (at the centroid of the analyzed segment).

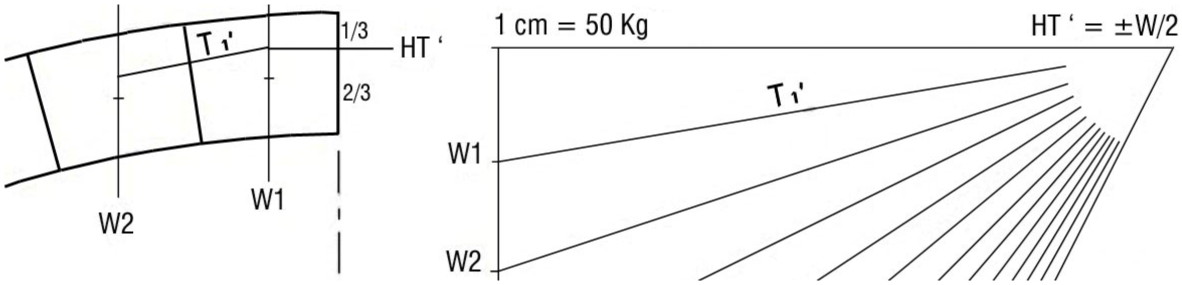


Fig. 45 – First resultant of the thrust

- Continue with the same procedure, transferring the resultant forces of the diagram onto the section of the arch. The various resultant forces will define a *provision line of thrust*, LT' – not the actual line of thrust.

- The final resultant force, T', is a provisional maximum thrust – not the actual maximum thrust.

- Extend the line of T' until it intersects the line of HT'. This intersection point, named I, is the "pivot of stability" of the arch, which closes the funicular diagram. A closed funicular diagram means that the arch is stable and in equilibrium.

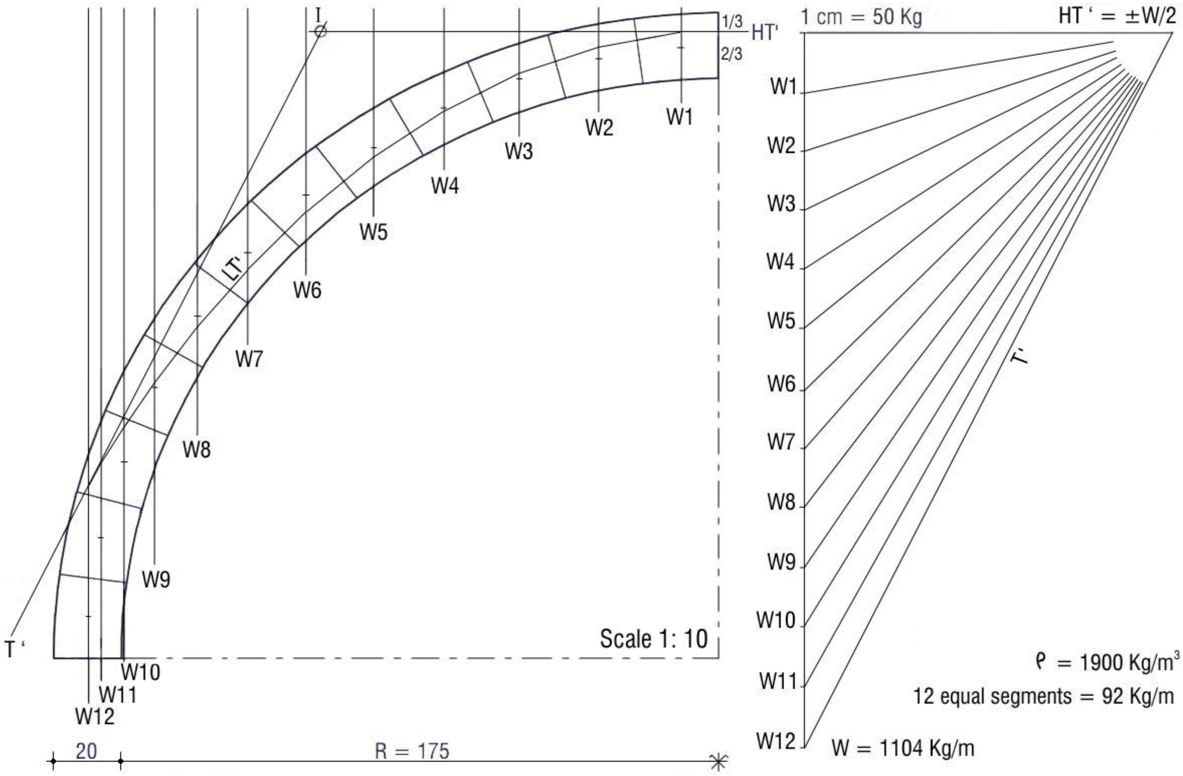


Fig. 46 – Resultant forces define LT' and I

10. Connect this point I and the ideal exit point of the thrust along the springer of the arch:
    Follow the pattern for typical arches in the table of Step 9. This will define the final direction of the thrust, T. Transfer T onto the funicular diagram (copy from the base of the diagram at the point of total weight). The intersection of T and HT' will define the actual horizontal thrust, HT, and will solve for all final thrust values.

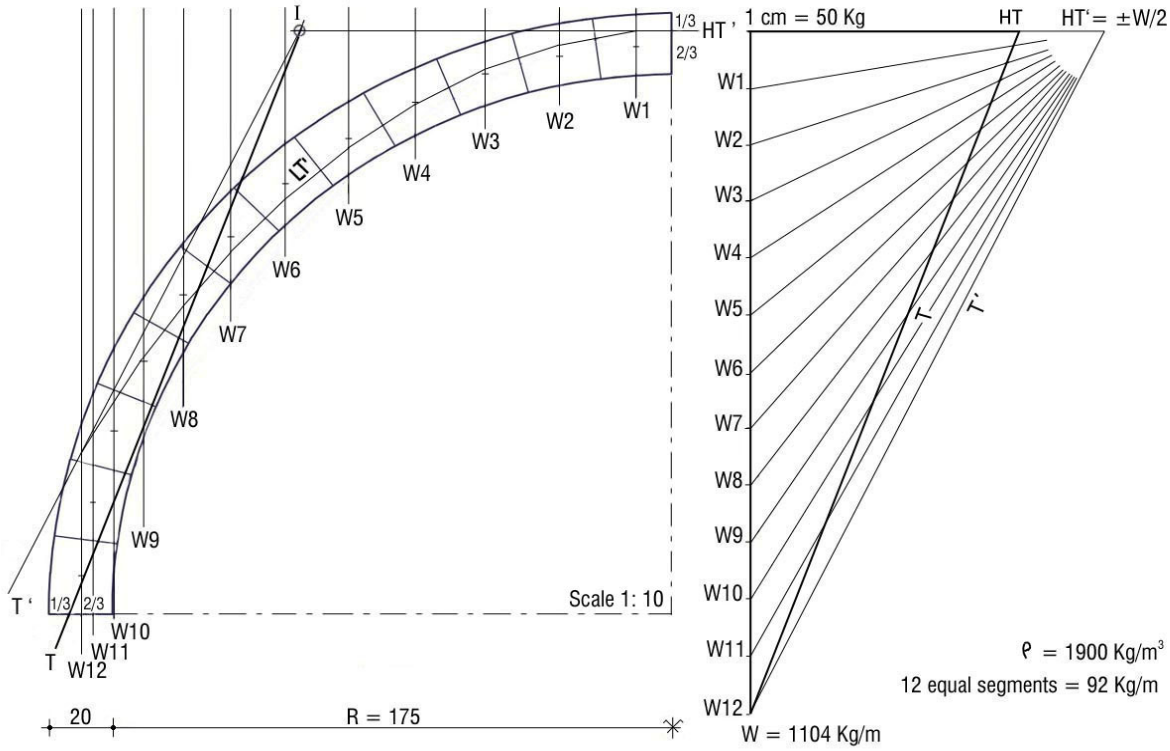


Fig. 47 – Final thrust of the arch

11. Draw all the resultant forces on the funicular diagram by joining HT and the various weights. This will define a new diagram which represents now the final forces of the funicular study.

    Study done by hand on a drawing table
    Only W is accurately known, as it was calculated at the beginning. T and HT cannot be calculated and are measured on the drawing.
    Thus it is essential to draw the diagram at the largest possible scale and as precisely as possible, so as to convert with the scale the magnitude with the minimum of error.
    As T and HT are not calculated, but approximated from the drawing, their magnitude should be indicated with ±.
    The value of T should be measured first as it is the longest force. In order to correct the inaccuracies of the hand drawing and to insure the equilibrium of the solution, the magnitude of HT and T should be rounded or slightly adjusted so that the triangle of the force polygon is closed with the Pythagorean theorem:
    $T^2 = W^2 + HT^2$

    Study done on computer
    The steps mentioned above are not needed as CAD programs provide accurate measurements.

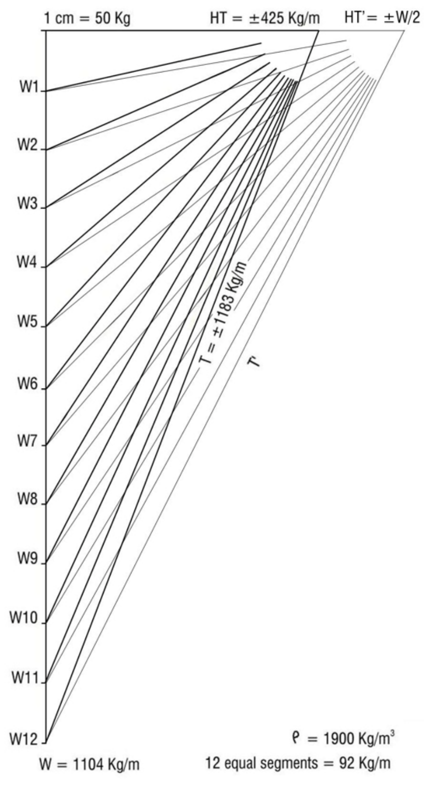


Fig. 48 – Final funicular diagram

12. Transfer all the resultant forces of the final funicular diagram onto the section of the arch. This will define the final solved-for line of thrust, LT.

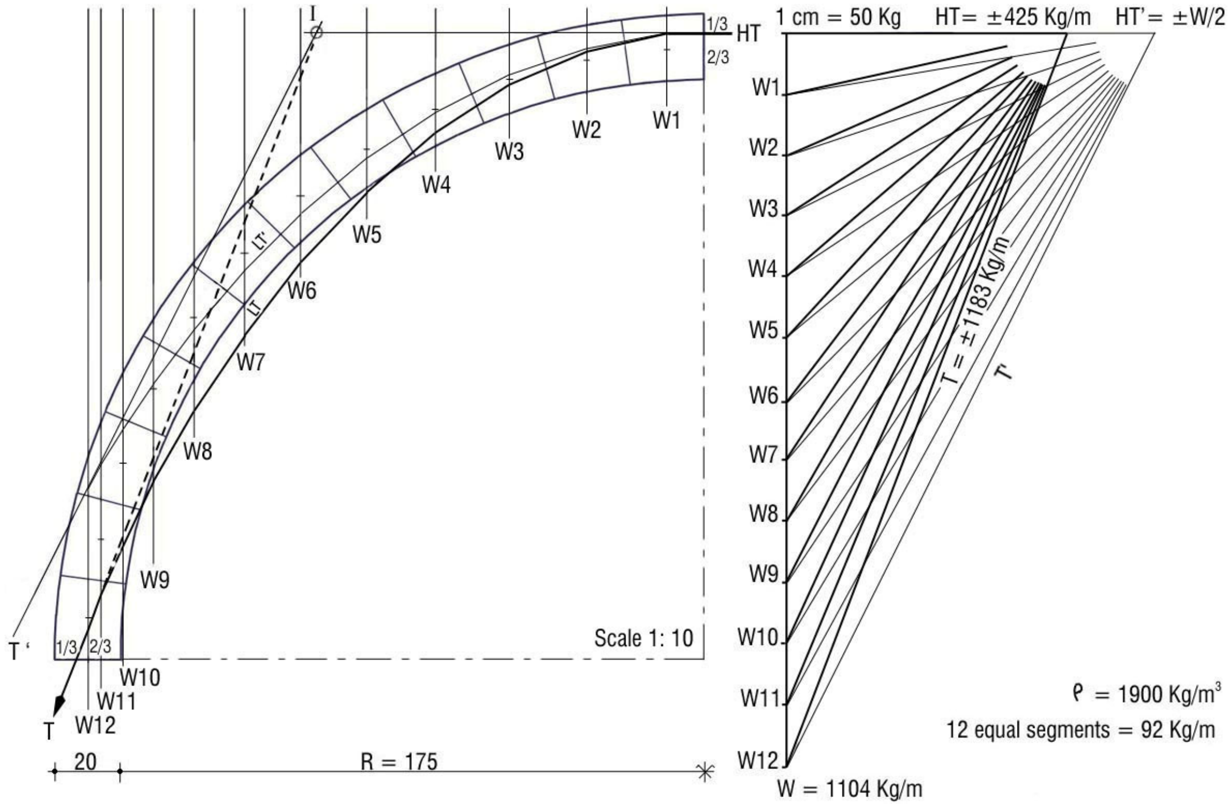


Fig. 49 – Final line of thrust

13. The line of thrust should remain within the middle third to ensure that the arch is both stable and safe.

    Note that the arch shown here as an example is not stable because it is too thin. The line of thrust passes outside of the cross-section of the material on the intrados side of the arch, so this arch will form a hinge on the intrados which will rotate outwards and cause collapse.

14. It is possible to make this arch, with corresponding span, stable by following the Optimisation method described in the following pages.

2.4 OPTIMISATION METHOD

2.4.1 Background & Aim

This method has been developed by the Auroville Earth Institute particularly for the design and construction of vaults without centring *(See Section 4.3: "Free Spanning" Technique, p. 87)*. The section of this optimal arch can also, of course, be used for building domes with no centrings. This method can also be applied for building an arch, though only in the case that the arch is not integrated into a wall. For an arch embedded within a wall, this optimisation method is unnecessary *(See Section 2.1.7.3: Arch Within a Wall, p. 24)*.

This method employs the Funicular method to design the optimal section of a desired arch. The optimal section will be defined as the lightest arch (e.g. the minimum-material arch), which contains the line of thrust within the middle third. The lighter the arch becomes, the less thrust it exerts.

Note that one approach to arch optimisation can be to target a reduction of horizontal thrust; however, this strategy will often lead to an increased total self-weight, due to the loading required on the haunches. In this case, the arch becomes heavier, and consequently, will exert more thrust even while its horizontal component has been reduced. As the reduction of the horizontal thrust is detrimental to the reduction of self-weight and overall thrust, it is preferable to target the goal of the lightness of the arch.

While in theory the ideal optimised arch is an arch with a catenary geometry (and thus of minimal thickness for a line of thrust to pass through the material), catenary arch geometries are much more complicated than regular geometries to accurately build. Formwork, guide-work or templates to control the arch geometry are often improperly fabricated, particularly in the case of a deep catenary arch, and inexperienced masons more commonly have difficulty accurately building these geometries.

2.4.2 Principle

Half of the arch to study is drawn with different thicknesses at top and bottom. The smallest thickness near the apex has to be defined. After several adjustments, the study will provide the ideal section, which is required to obtain the lightest arch, according to its span and shape. Once the lightest arch has been obtained with the Funicular method, the masonry pattern can be determined.

2.4.3 Method

The method begins with the Funicular study and continues thereafter with the study of the masonry bond pattern.

1. <u>Defining and drawing the arch section</u>

1.1 - Draw half of the desired arch using a large scale (1/5 to 1/10) which fits on the tracing paper.
- If a computer is used, draw half the arch in centimetres on the model space.
- Define the arch's bottom thickness in relation to the span:
  - ➢ 1/12 for 3 to 6 m span = ~ 25 to 30 cm
  - ➢ 1/16 for 6 to 10 m span = ~ 30 to 35 cm
  - ➢ 1/30 for 10 to 15 m span = ~ 35 to 40 cm
  - ➢ 1/50 for 15 to 25 m span = ~ 40 to 50 cm
- Define the minimum thickness at the top of the arch:
  - ➢ 7 cm for spans between 3 to 6 m
  - ➢ 9 cm for spans between 6 to 10 m
  - ➢ 11.5 cm for spans between 10 to 15 m
  - ➢ 21 cm for spans between 15 to 25 m

- This minimum thickness will depend on the block size available. The minimum thickness with the Auram Mini blocks is 7 cm.
- Once the thicknesses have been defined, draw the centreline and the middle third of the arch.

1.2 - Calculate the angles of the various radiuses, if the arch is segmental or has several centres.
- Calculate the average thickness (t) of the arch = (Bottom thickness + top thickness)/2
- Calculate the length (in m) of the arch centre line:

$$A_{Centreline} = \frac{\pi(R + 1/2t)\alpha}{180}$$

Where: R = Intrados radius (m), t = average thickness (m), α = angle of the arch
- Note that the centreline of the arch will not correspond with the line of thrust; nevertheless, this approximation is sufficient for the study.

2. <u>Funicular study</u>

2.1 - Start the funicular study as described in Section 2.3: Funicular Method, p. 30.

2.2 - Note that the arch has now different thicknesses from top to bottom.
- LT should enter in a different way for some arches according to the following pattern:

Arch type	Entry of LT	Exit of LT
➢ Segmental arches	Centre of the arch	Centre of the arch
➢ Bucket arches	Now at the centre of the arch	At 2/3 from the arch intrados
➢ Semicircular arches	Now at the centre of the arch	At 2/3 from the arch intrados
➢ Egyptian arches	Now at 1/3 from the arch intrados	At 2/3 from the arch intrados
➢ Catenary arches	Centre of the arch	Centre of the arch
➢ Equilateral arches	At the intrados of the arch	At 2/3 from the arch intrados
➢ Pointed arches	In the intrados third of the arch	At 2/3 from the arch intrados
➢ Corbelled arches	Centre of the arch	Centre of the arch

2.3 The first diagram may not determine a LT within the middle third.

2.4 - The thickness has to be adjusted along the extrados curve and/or at the bottom of the arch.
- Adjust the segment width:
- Increase the thickness, and thus the weight, where LT is towards the intrados.
- Decrease the thickness where LT is towards the extrados.
  - ⇒ <u>In many cases, when LT is not in the middle third at a given position, the problem has to be solved either before or after this position, because the arch is either too thin or too thick elsewhere on the arch.</u>
  - ⇒ <u>Thickness is minimal near the apex. Therefore, if LT is close to the extrados at the upper portion, the thickness should not be reduced but kept as such and sometimes even increased, according to the arch type (i.e. Egyptian arch).</u>
- Note that increasing the bottom thickness may increase the total weight of the arch and therefore will not necessarily provide the most optimised arch.
- It is better to try optimising the thickness along the extrados curve and only in the last resort to increase the bottom thickness.
- Calculate the new weights and centres of gravities for all segments, and repeat the stability study with the funicular method to check if the line of thrust is now within the middle third.

2.5 - It might be necessary to repeat the previous step (2.4) a few times:
- Adjust the thickness of some segments.
- Calculate their weights.
- Repeat the funicular diagram until LT remains within the middle third.

2.6 - It is necessary sometimes to move the exit of LT, in order to position it within the middle third of the arch.

2.7 - Point 2.2 mentions that LT exits in a certain way, depending on the type of arch.
- This condition has been obtained by extensive empirical research and it is a safe limit. However, this can be changed: Fig. 50 shows that LT 1 exits as suggested, but it exits the middle third towards the top of the arch. Elsewhere it is safely in the middle third.
- The exit of LT can be changed (as per LT 2) and then the line of thrust remains everywhere in the middle third of the arch.

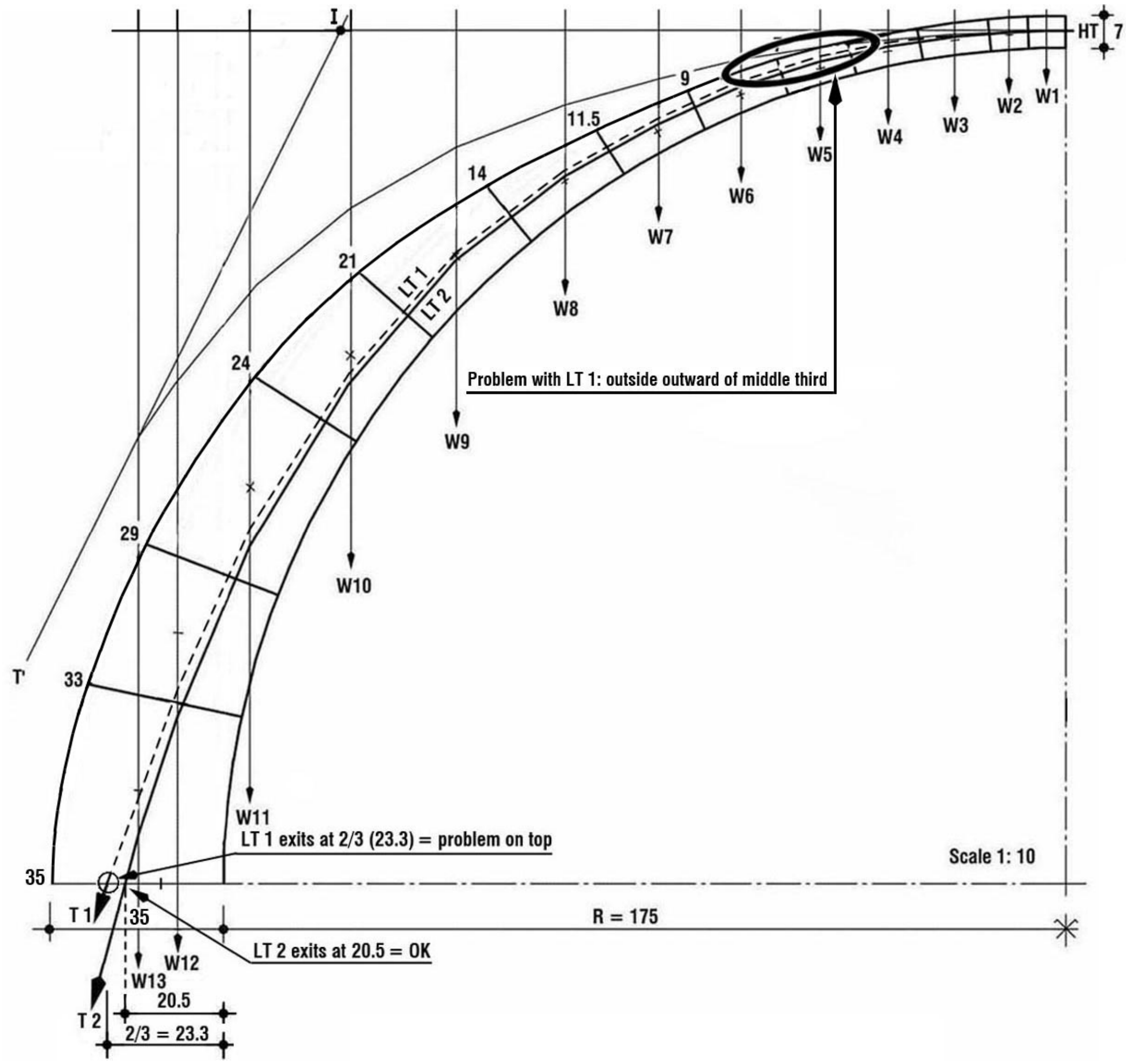


Fig. 50 – Changing the exit of LT

2.8 This principle can also be applied with the entry of LT. By moving the entry of LT a little bit up or down from the theoretical entry, it may be possible to determine a LT which is everywhere within the middle third.

2.9 - Once LT is within the middle third, the arch is stable but it could still be optimised further.
- The aim is now to minimize the thickness of the bottom of the arch.
- The minimum thickness on the springer is obtained when LT touches the limits of the middle third approximately three times, while remaining within it. When this is achieved, the arch is as light as possible. It has been fully optimised and the stability study is complete.
- Note that this step is optional, as the arch was already stable.

3. Checking for Crushing

3.1 Now that the stability study is complete, a check needs to be done to ensure that there is no crushing in the vault (i.e. that the maximum stress in the vault does not exceed the crushing strength of the material).

3.2 - The wet compressive strength of the blocks should be considered, as this is the weakest state of stabilized earth material.
- Normally, stress is greatest at the base of an AVD, where the maximum forces are. However, with the optimisation method, the maximum stress is often towards the apex, as the thickness is minimal.
- A safety factor of 5 should be kept from the wet crushing strength for the admissible load bearing: Load bearing $\leq$ Wet crushing strength / 5

- Example:
The wet crushing strength of CSEB is 20 kg/cm^2. Nowhere in the arch should the load bearing exceed 4 kg/cm^2 (20/5).

- Calculating the load bearing:
HT = 270 kg/m – Mini thickness = 7 cm – Wet crushing strength of a CSEB block = 20 kg/cm^2
$\Rightarrow$ Admissible load bearing = 20 / 5 = 4 kg/cm^2
$\Rightarrow$ Area per running meter at the apex = 7 x 100 = 700 cm^2
$\Rightarrow$ Load bearing = HT / Area at the apex = 270 / 700 = 0.38 kg/cm^2
$\Rightarrow$ Load bearing is safe as 0.38 $\leq$ 4

Until now, only the stability of the arch has been considered. All segments have theoretical dimensions. The arch must now be studied with real block sizes and masonry pattern.

4. Masonry study

4.1 - Determination up to which height the vault can be built with horizontal courses.
- If the purpose of the study is only to build an arch and not a vault, this step and the following steps (until the end of the optimisation method) do not need to be followed. They should be followed only when a vault is to be built with the AVEI Free Spanning technique.

- Trace the line for the limit of stability which passes at the point from the inside of the springer.

- Calculate the moments of segments on either side of the stability limit.

- The sum of the moments of the segments located on the left side of the stability limit should be equal or greater to the sum of those located on the right side:

$$\sum(W \cdot X)\text{Left side} \geq \sum(W \cdot X)\text{Right side}$$

Where: W= Weights (kg)
X= Dimensions (m) from the limit of stability

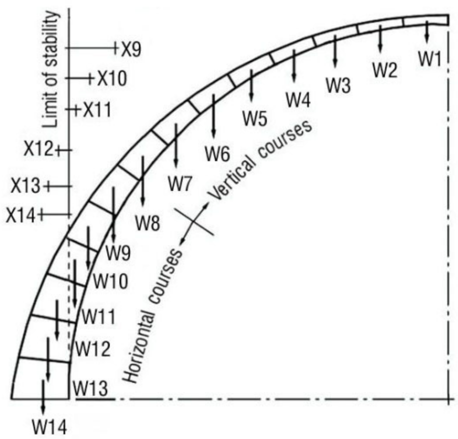


Fig. 51 – Calculating moments for a vault

- Fig. 51 shows:
$(W_{14} \cdot X_{14}) + (W_{13} \cdot X_{13}) + (W_{12} \cdot X_{12}) \geq (W_{11} \cdot X_{11}) + (W_{10} \cdot X_{10}) + (W_9 \cdot X_9)$
Thus this vault can be built with horizontal courses up to segment W9.

4.2 - Define the height of blocks which will be used for the horizontal courses.
- If Auram blocks are to be used, it will preferably be 9 cm high.
- The thickness of blocks used for the vertical courses will imperatively be 5 cm thick.

4.3 - Divide the total height of segments to be built with horizontal courses by the chosen block height.

4.4 - For both horizontal and vertical courses, define the block sizes which will fit the decreasing thickness of the arch *(See Table: Blocks made by the Auram Press 3000, p. 43)*.

4.5 **- It is essential that neither the extrados or intrados of the arch profile is modified at this stage; otherwise the stability will change and the entire stability study will have to be redone.**

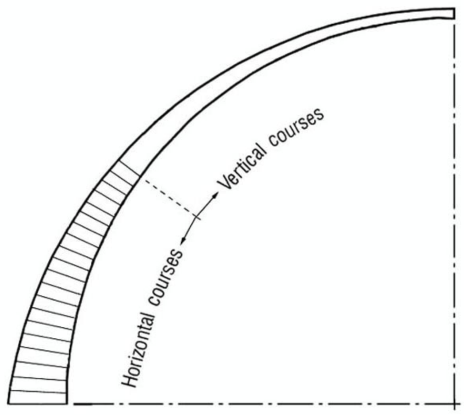


Fig. 52 – Division of segments by block height

4.6 - Define the masonry pattern:
- Blocks of similar sizes should be used for several courses, in order to establish good bonds.

- Steps between various courses should be kept to the minimum, as they are detrimental to the strength of the arch.

- Integrating the block sizes and the bond pattern may require slightly adjusting the length of some segments.

4.7 - The triangular region between the extrados curve and the block courses, which will be filled with an earth concrete, should be kept to the minimum.

- The volumic mass of the earth concrete used for the filling should be known *(See Section 4.4: Binder Quality, p.90)*, and it should preferably be close to the volumic mass of the blocks.

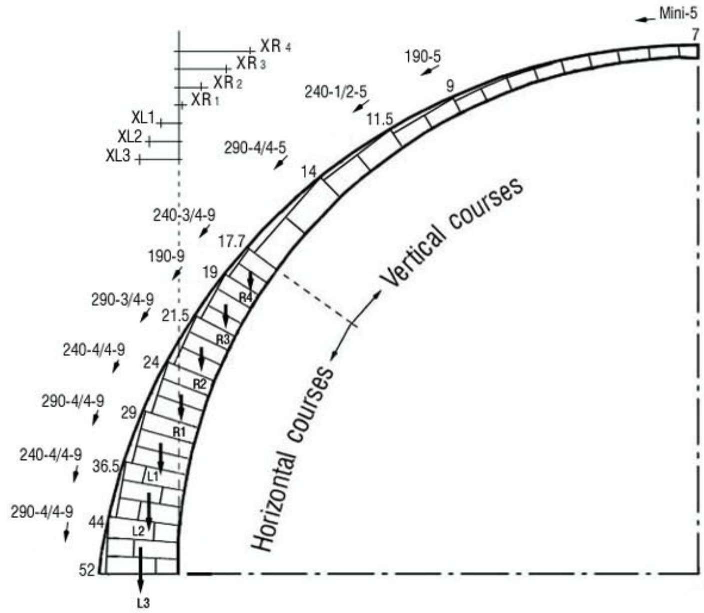


Fig. 53 – Define the block sizes and masonry pattern

4.8 - While defining the block sizes and the bond pattern, the previous segments used for analysis are modified.
- Check if the top level of the horizontal courses has not changed too much.

4.9 - If the top level of the horizontal courses has changed, it is necessary to redo the calculation of the moments with the new segments of the masonry pattern.
- This has to be done only if the horizontal courses of the masonry are higher than they were initially calculated. The left side moments should never be less than the right side:

$$\sum(W \cdot X)\text{Left side} \geq \sum(W \cdot X)\text{Right side} \quad \text{(kg m)}$$

- Fig. 53 shows that: $(L_3 \cdot XL_3) + (L_2 \cdot XL_2) + (L_1 \cdot XL_1) \geq (R_1 \cdot XR_1) + (R_2 \cdot XR_2) + (R_3 \cdot XR_3) + (R_4 \cdot XR_4)$

- L_1, R_1, etc. are the reference for the weights (kg) and XL_1, XR_1, etc. are the reference for the dimensions (m) between the axis of the CGs of L_1, R_1, and the limit of stability.

4.10 - Once the top level of all the horizontal courses is sure, define for each principle course the following dimensions, in order to be able to check on site that the arch is rising properly:

1. Arch intrados, the length along the intrados from the springer line to the top of the considered course:
$A_{Intrados}\,(\text{cm}) = \text{Number of blocks} \times \text{Block height (cm)}$

2. Angle (with 2 decimals) from the springer line to the top of the considered course:

$$\alpha = \frac{180 \cdot \text{Intrados}}{\pi R}$$

3. Cord from the springer line to the top level of the considered course: $\text{Cord(cm)} = 2R\sin\left(\frac{\alpha}{2}\right)$

4. Span at the top of the considered course:
Semicircular arch $\Rightarrow Span = 2R\cos\alpha$ Pointed arch $\Rightarrow Span = 2[(R\cos\alpha) - E]$

Where: R = Radius (cm), E= Eccentricity of a pointed arch (cm)

5. Height at the top level of the considered course: $\text{Height(cm)} = R\sin\alpha$

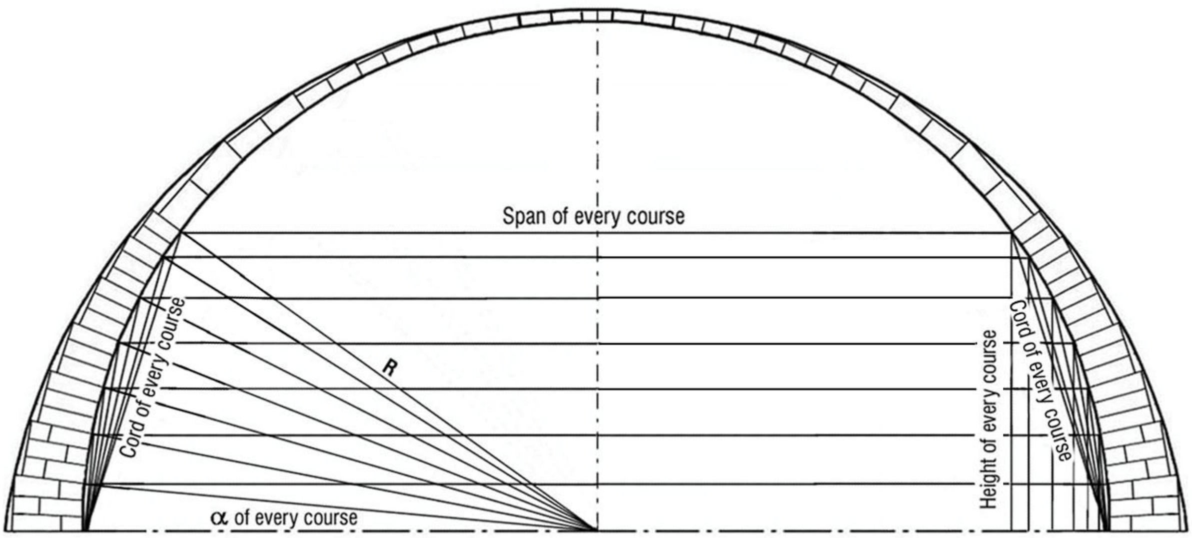


Fig. 54 – Dimension the cord, span, height and angle of principle horizontal courses

4.11 - Reference all block sizes *(Fig. 56)*. Dimension the thickness of the arch, if it is not given by the block size. Label all information required to execute the work on site.

4.12 - The arch section has now been optimised. The entire study is over and the construction can begin.

4.13 - If a vault is built with a combination of horizontal and vertical courses, check regularly how the vault rises. The cord and span measured on site for each new course should not vary too much from the calculation.

4.14 - The essential parameter to maintain is the thickness of the vault at a determined height.
- Some site adjustment might be needed.

Blocks made by the Auram Press 3000

The Auram press 3000 can produce a wide variety of blocks. These blocks can be used for single, one and a half, or double bond pattern.

According to how the bond pattern is organised, these thicknesses can be obtained with the blocks made by the Auram press 3000:

- 58 cm (double bond pattern with the block 290)
- 49 cm (double bond pattern with the block 240)
- 44 cm (one and a half bond pattern with the block 290)
- 39 cm (single bond pattern with the full-size hollow block 390 laid in header)
- 36.5 cm (one and a half bond pattern with the block 240)
- 29 cm (single bond pattern with the full-size block 290 laid in header)
- 24 cm (single bond pattern with the full-size block 240)
- 21.5 cm (single bond pattern with the ¾ block 290 laid in header)
- 19 cm (single bond pattern with the block 190 laid in header or the block 390 laid in stretcher)
- 17.7 cm (single bond pattern with the ¾ block 240 laid in stretcher)
- 14 cm (single bond pattern with the full block 290 laid in stretcher)
- 11.5 cm (single bond pattern with the ½ block 240 laid in stretcher)
- 9 cm (single bond pattern with the block 190 laid in stretcher)
- 7 cm (single bond pattern with the Mini block 290 laid in stretcher)

Block name	Reference	Nominal block size (L x W x H) in cm
390	390 – 4/4 – 9 (Full size) 390 – 3/4 – 9 (3/4 size) 390 – 1/2 – 9 (1/2 size)	39 x 19 x 9 * 29 x 19 x 9 * 19 x 19 x 9 *
290	290 – 4/4 – 9 (Full size) 290 – 3/4 – 9 (3/4 size) 290 – 1/2 – 9 (1/2 size)	29 x 14 x 9 * 21.5 x 14 x 9 * 14 x 14 x 9 *
240	240 – 4/4 – 9 (Full size) 240 – 3/4 – 9 (3/4 size) 240 – 1/2 – 9 (1/2 size)	24 x 24 x 9 * 24 x 17.7 x 9 * 24 x 11.5 x 9 *
190	190 – 9 (Full size)	19 x 9 x 9 *
Mini 290	Mini – 5 (Full size)	14 x 7 x 5

Table 1 – Blocks made by the Auram Press 3000

Notes:

* The nominal block height of the Auram blocks is 9 cm, but the block height can vary from 5 to 10 cm, from millimetre to millimetre.

If the chosen block height is different than 9 cm, the dimensions indicated in the above table should be changed accordingly. Remember that the vertical courses should imperatively use blocks of 5cm thick, in order to maintain a low weight per surface area to insure the adhesion of the block on the vaulted surface.

2.4.4 Presentation of the Study

The drawings for the optimisation study should be drafted as shown in the following examples.

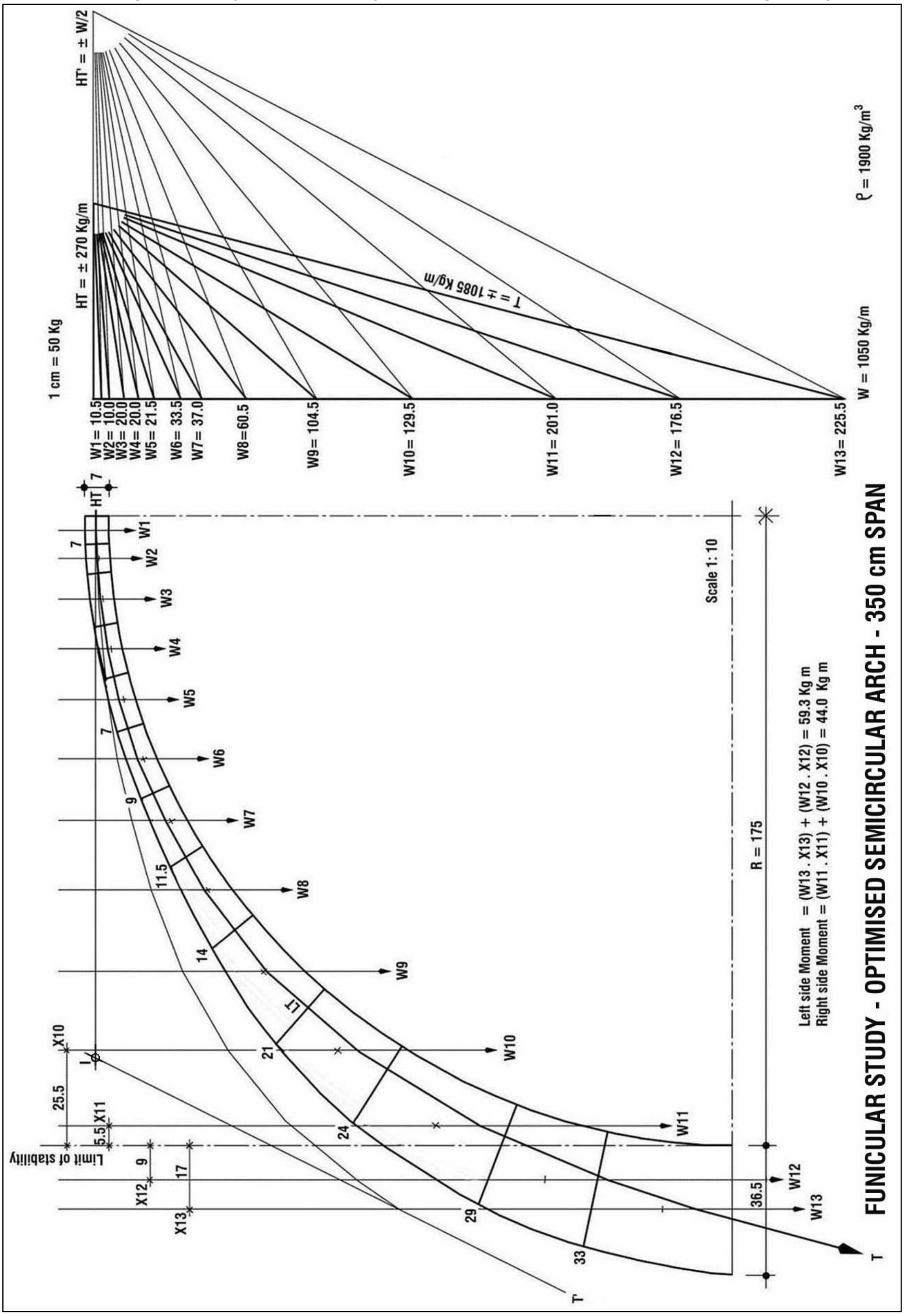


Fig. 55 – Presentation of the funicular study

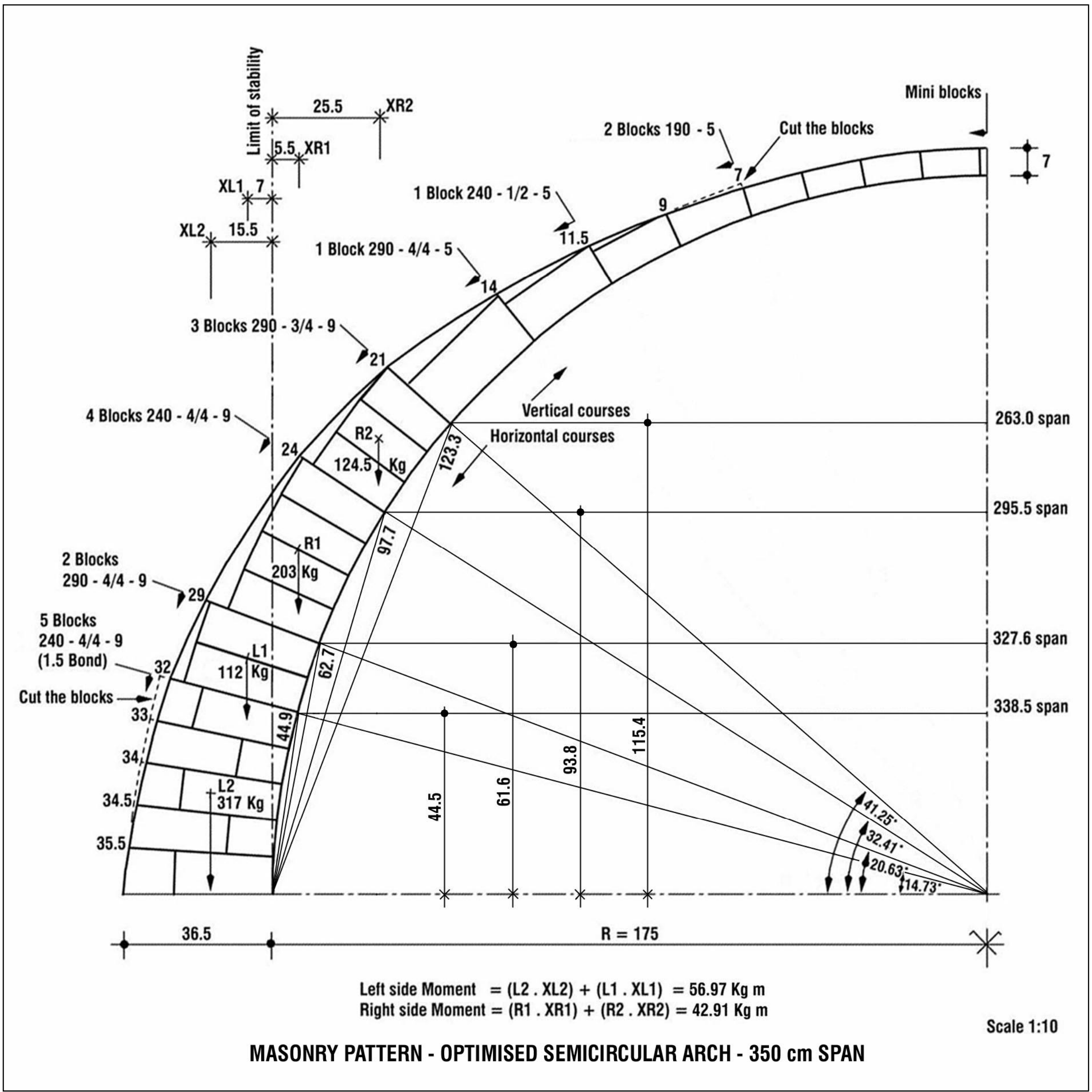


Fig. 56 – Presentation of the masonry pattern

Label all information required to execute the work on site.

2.5 FUNICULAR STUDIES OF TYPICAL ARCHES

The following funicular studies demonstrate the different behaviour of typical arches of various thicknesses.

Several arches are stable (Segmental, Catenary and Corbelled), but the others (Bucket, Semicircular, Egyptian, equilateral) are not stable and require some modification to optimise their cross section.

The optimisation of the cross section can be done with the Optimisation method. Note how the line of thrust (LT) enters, exits and moves within the arch:

- **Segmental arches** :
  - LT enters and exits in the centre.
  - LT is always close to the centre.
  - The flatter the arch is, the closer LT will be to the centre.
  - When the arch is more rounded and the rise increases, LT will be closer to the intrados of the middle third.

- **Bucket arches** :
  - LT enters at 2/3 and exits at 2/3 from the arch intrados.
  - LT exits the arch at the intrados, with a maximum difference at the level of the haunches.
  - When the thickness increases a lot (for the same span), it will remain within the intrados third, but not within the middle third.

- **Semicircular arches** :
  - LT enters at 2/3 and exits at 2/3 from the arch intrados.
  - Depending upon the thickness, it will exit the arch at the intrados, or will remain within the intrados third at the level of the haunches.
  - LT will remain within the middle third only when: $T \geq S / 5$

- **Egyptian arches** :
  - LT enters at 2/3 and exits at 2/3 from the arch intrados.
  - Depending upon the thickness, it will exit the arch at the intrados, or will remain within the intrados third at the level of the haunches.
  - LT will remain within the middle third only when: $T \geq S / 7$

- **Catenary arches** :
  - LT enters and exits in the centre.
  - LT is always centred within the arch, whatever the proportions of the catenary geometry are.

- **Equilateral arches** :
  - LT enters at the intrados and exits at 2/3 from the arch intrados.
  - The weight of the keystone is essential for the stability of this arch.
  - Depending upon the thickness:
    - LT goes close to the extrados (or even exits the arch), near the top of the arch.
    - LT will exit the arch (intrados side) or near the intrados at the level of the haunches. It will remain within the intrados third only when the arch thickness increases significantly.
    - LT will remain within the middle third only with a tremendous arch thickness.

- **Corbelled arches** :
  - LT enters and exits in the centre.

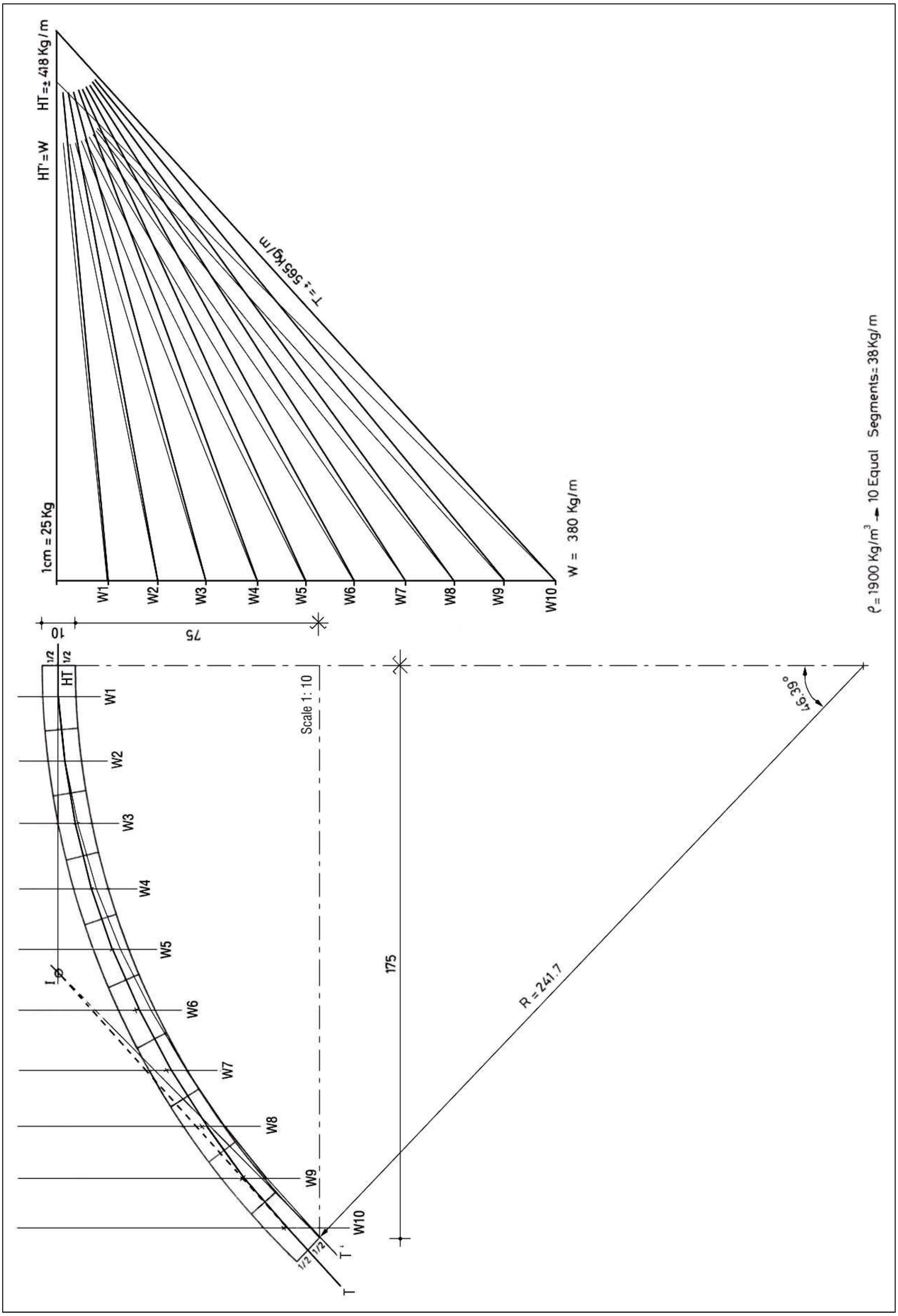


Fig. 57 – Segmental arch, 350 cm span, 75 cm rise, 10 cm thick

This arch is stable: LT is close to the centre of the arch.

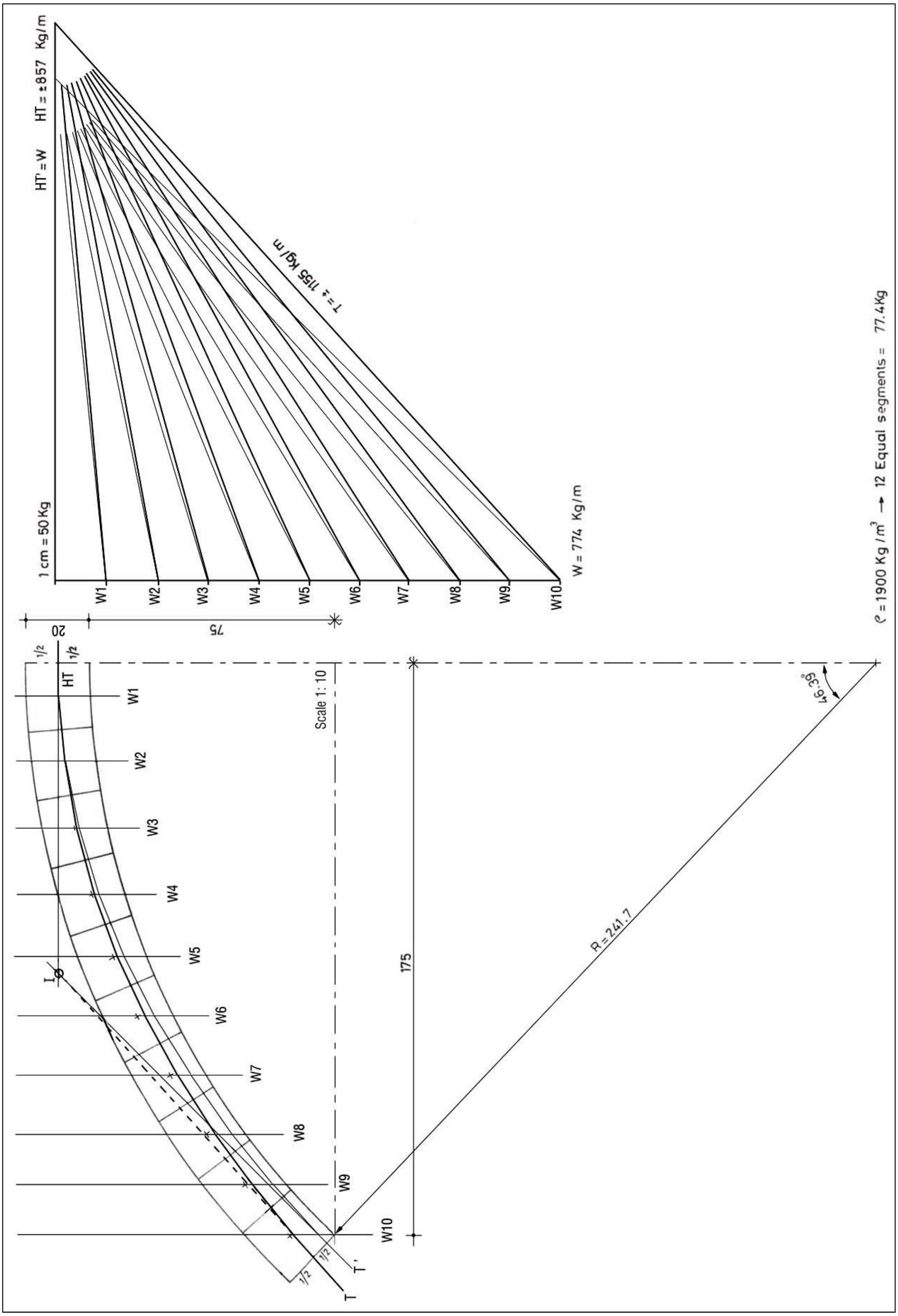


Fig. 58 – Segmental arch, 350 cm span, 75 cm rise, 20 cm thick

This arch is stable: LT is close to the centre of the arch.

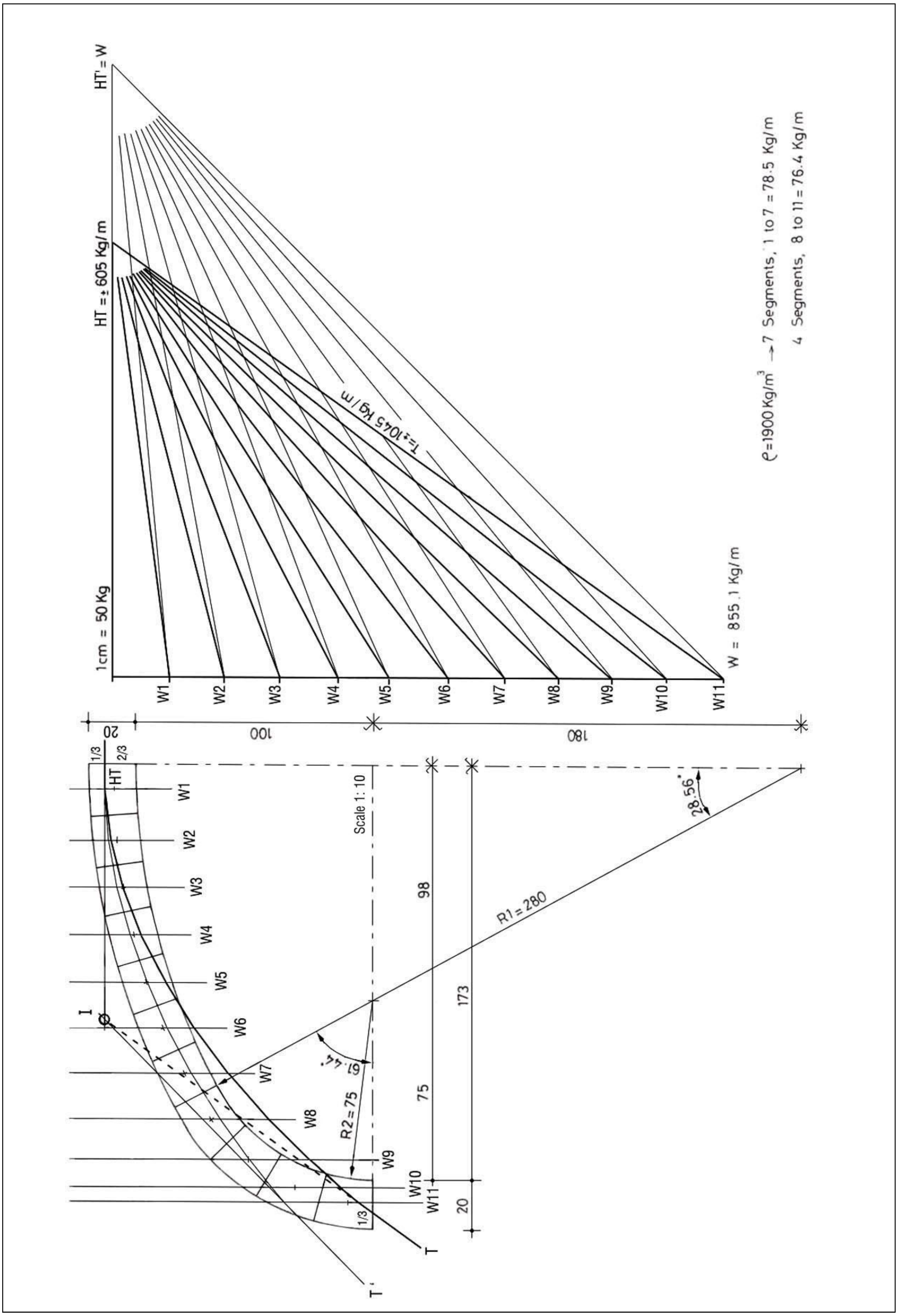


Fig. 59 – Bucket arch, 346 cm span, 100 cm rise, 20 cm thick

This arch is not stable: LT exits the arch.

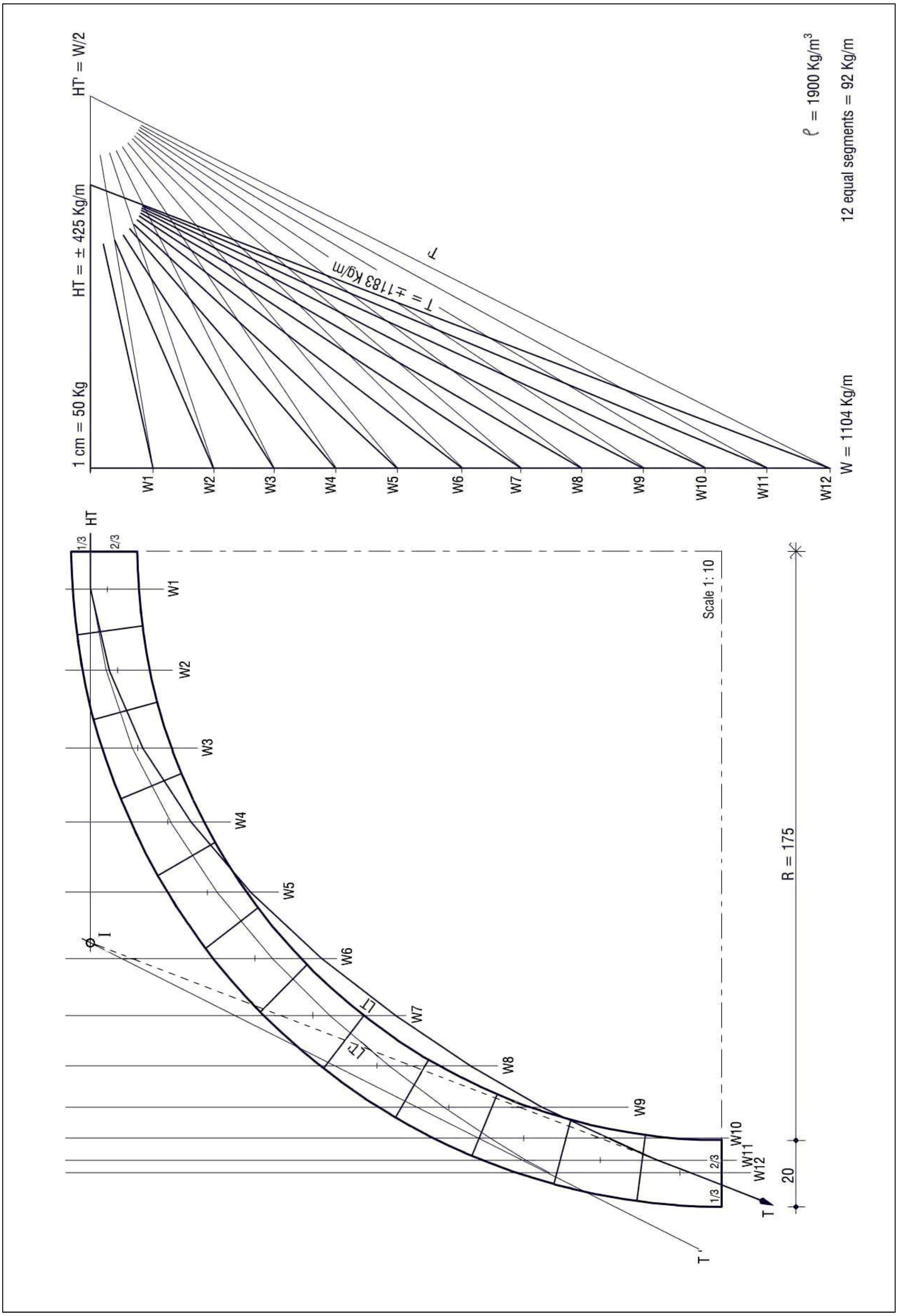


Fig. 60 – Semicircular arch, 350 cm span, 20 cm thick

This arch is too thin and not stable (t = S / 17.5). It requires a thickness of 70 cm to be stable.

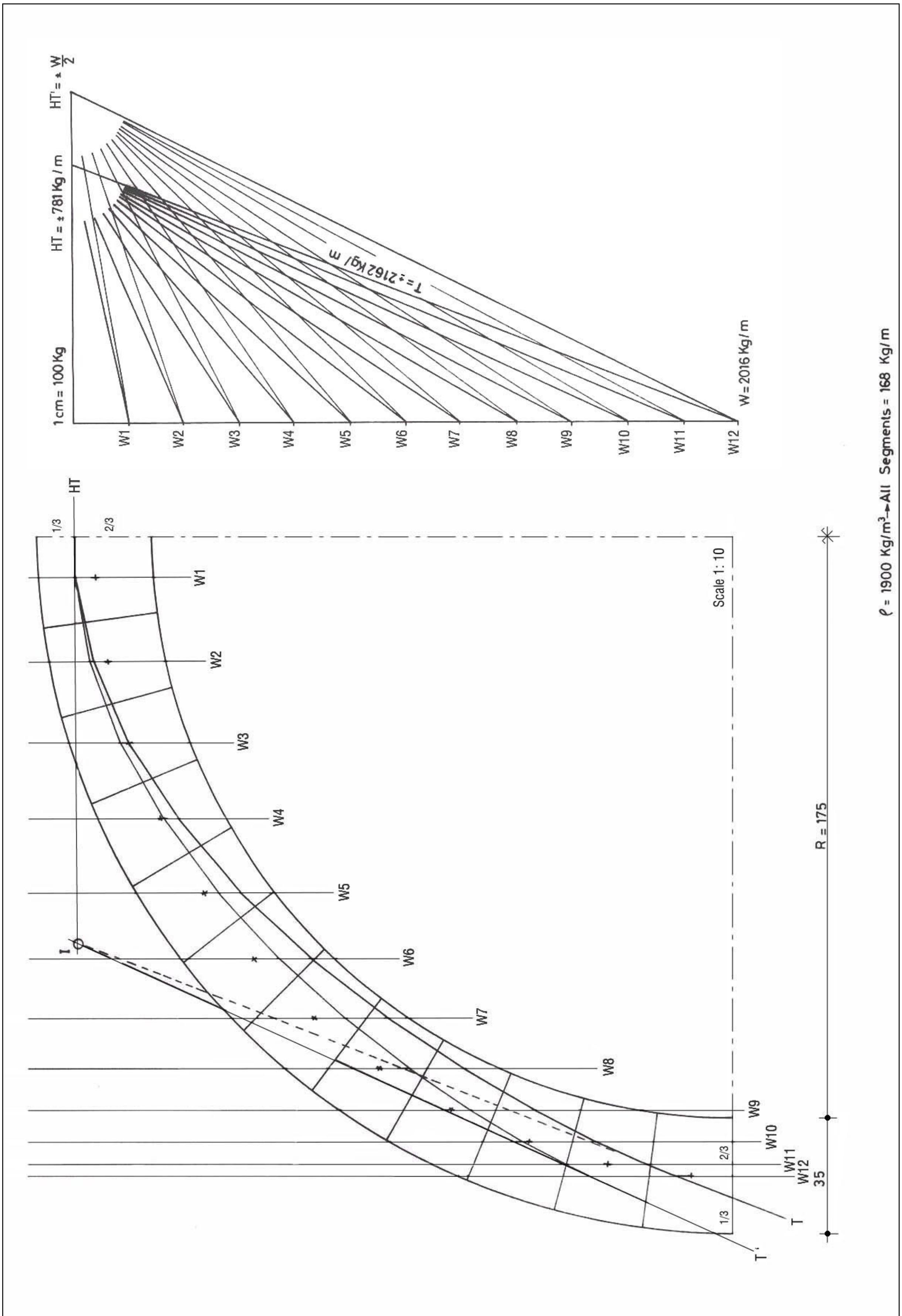


Fig. 61 – Semicircular arch, 350 cm span, 35 cm thick

This arch is too thin and not stable (t = S / 10). It requires a thickness of 70 cm to be stable.

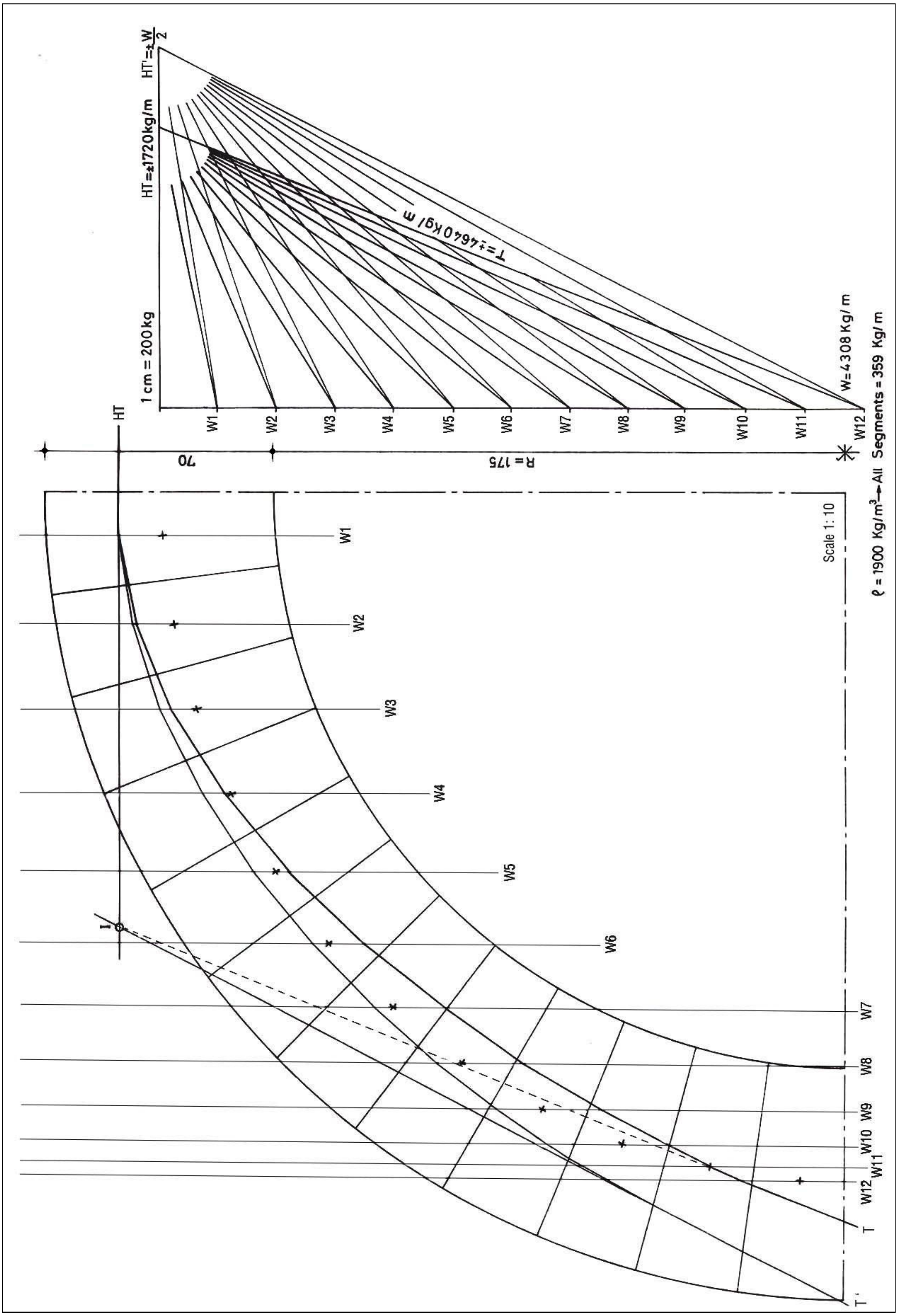


Fig. 62 – Semicircular arch, 350 cm span, 70 cm thick

This arch is thick enough and stable (t = S / 5).

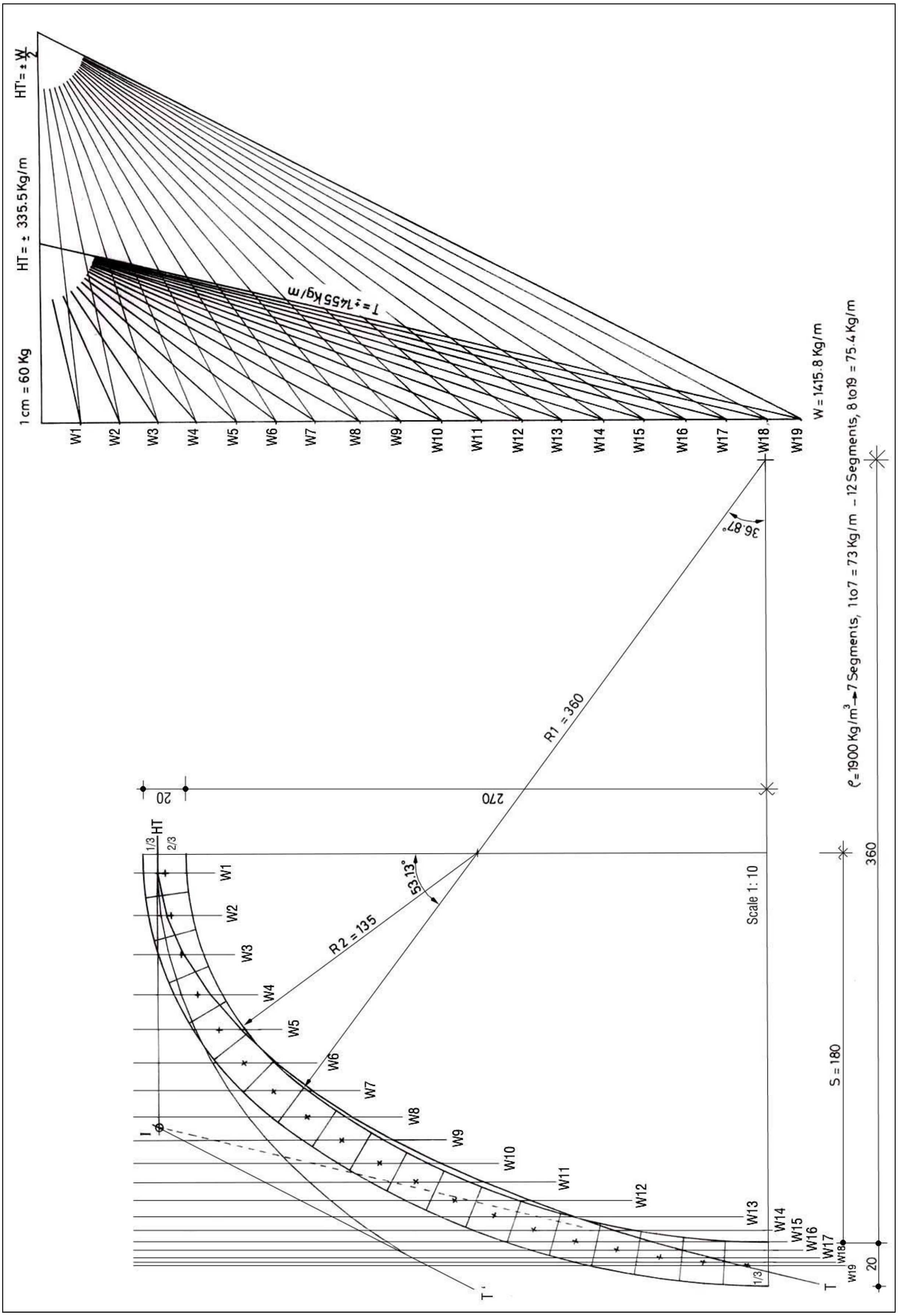


Fig. 63 – Egyptian arch, 360 cm span, 270 cm rise, 20 cm thick

This arch is too thin and not stable (t = S / 18): It requires a thickness of 51.4 cm to be stable.

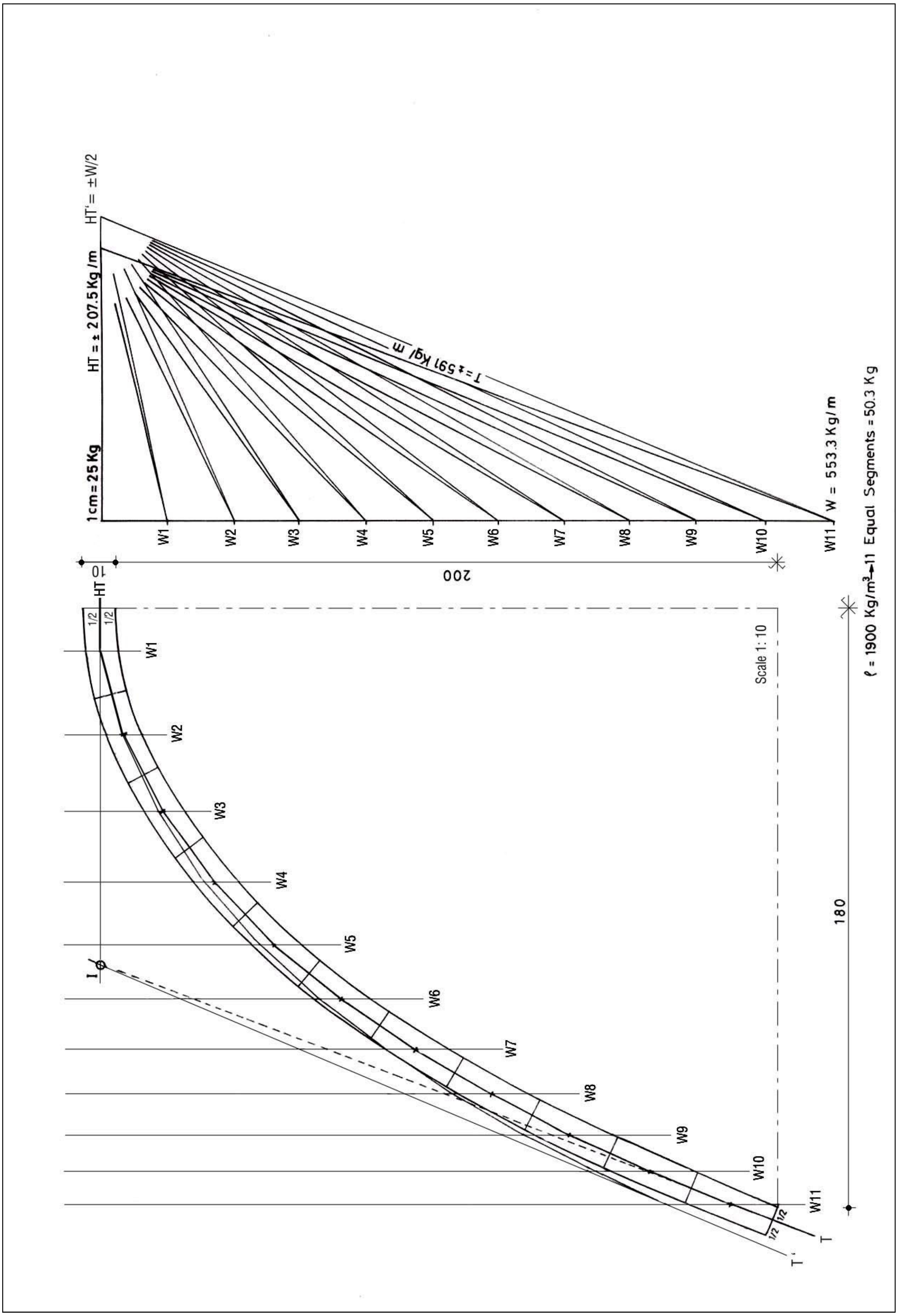


Fig. 64 – Catenary arch, 360 cm span, 200 cm rise, 10 cm thick

This arch is the most stable: LT is centred within the arch.

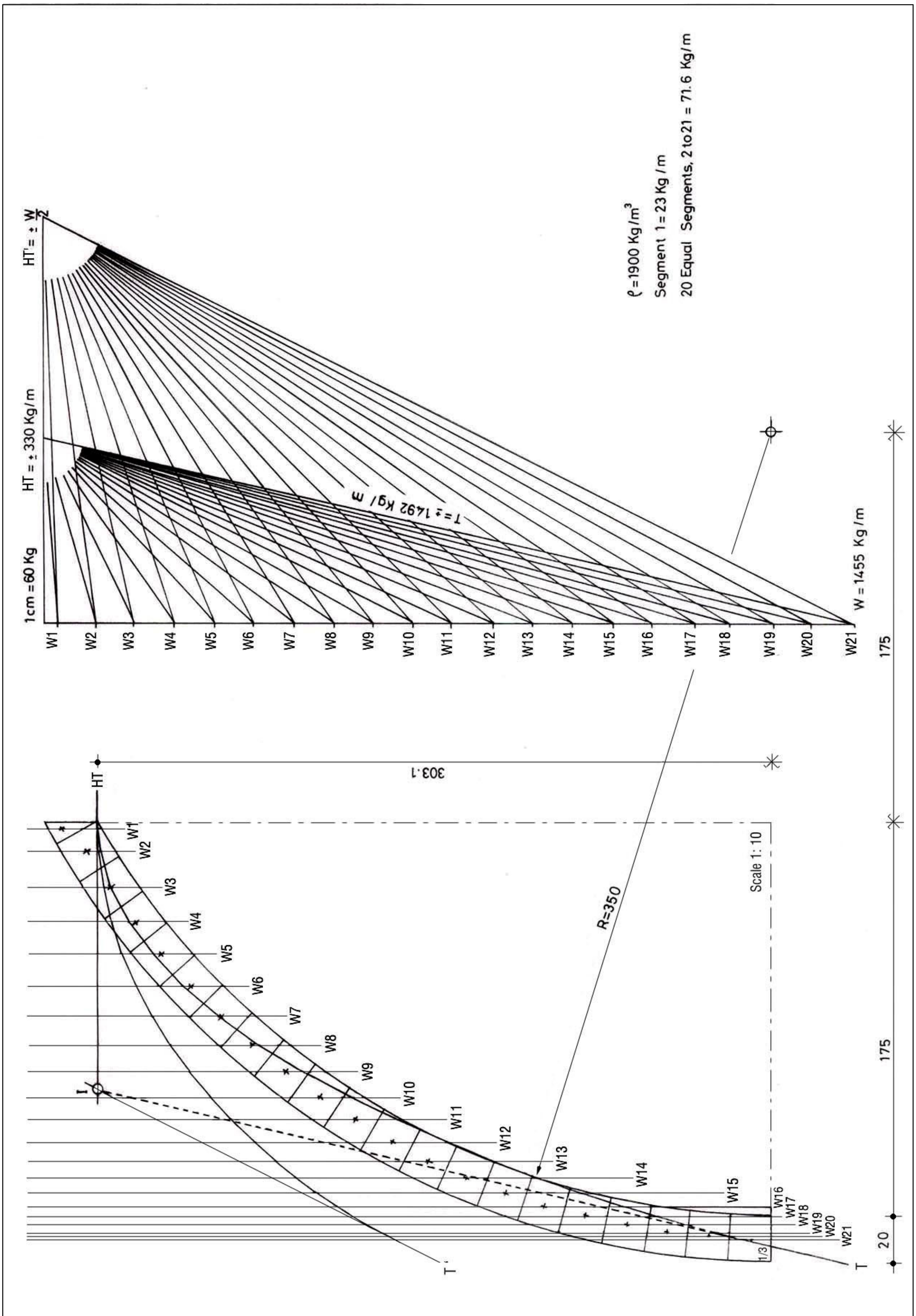


Fig. 65 – Equilateral arch, 350 cm span, 303.1 cm rise, 20 cm thick

This arch is stable but not safe: LT does not remain within the middle third of the arch.

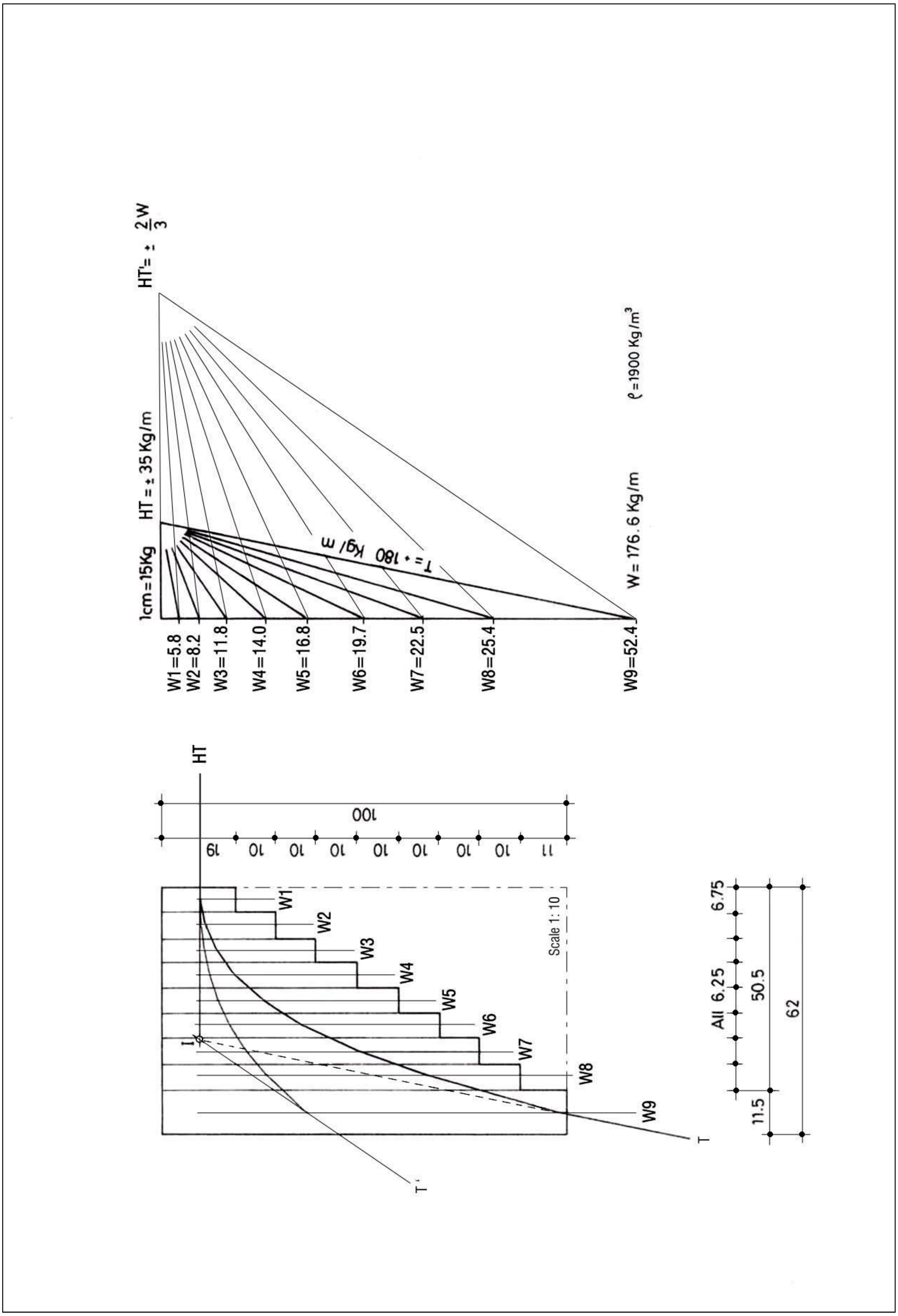


Fig. 66 – Corbelled arch, 101 cm span, 81 cm rise, 24 cm wide, pier = 11.5 cm

This arch is stable: LT remains within the arch.

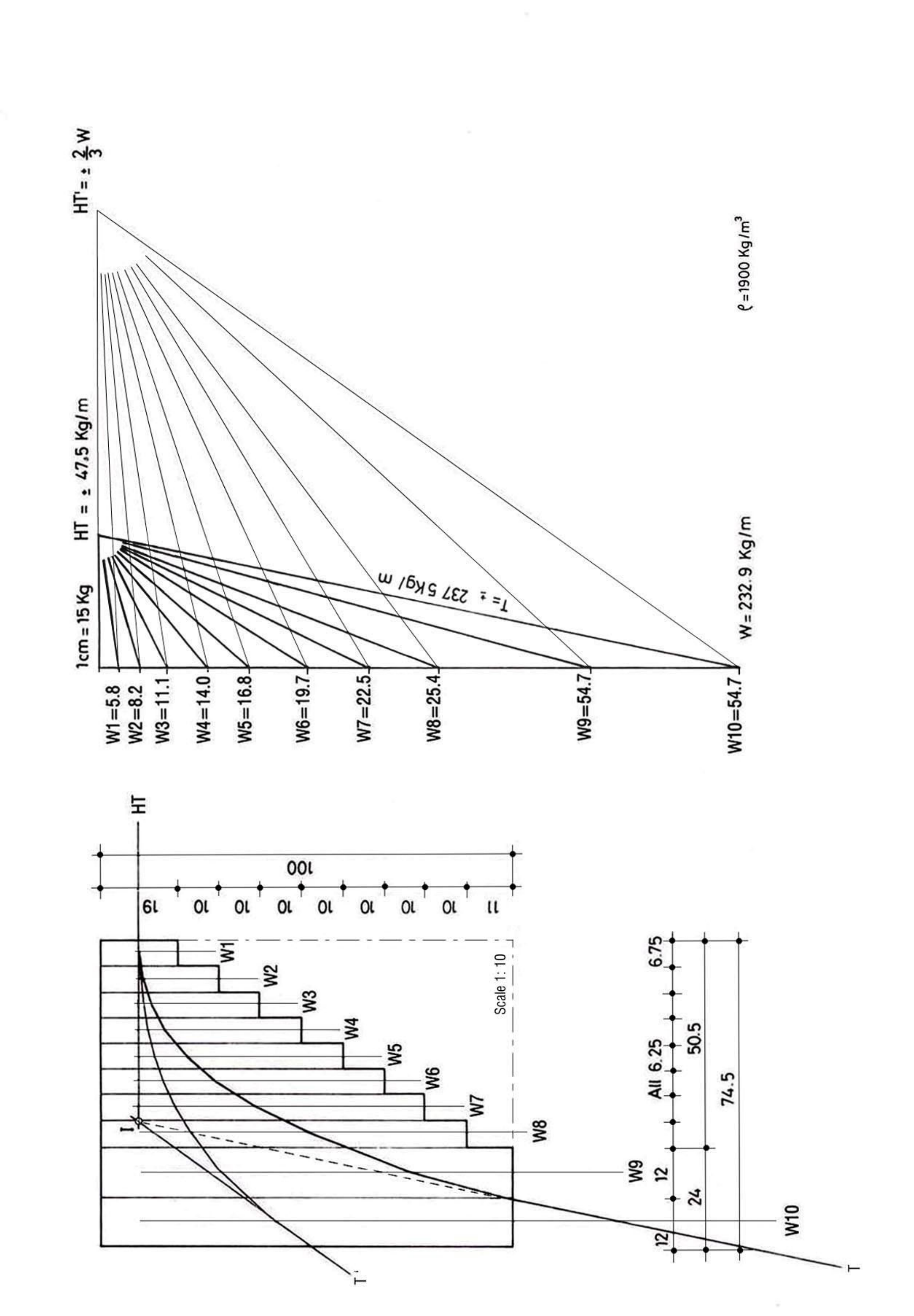


Fig. 67 – Corbelled arch, 101 cm span, 81 cm rise, 24 cm wide, pier = 24 cm

This arch is stable: LT remains within the arch.

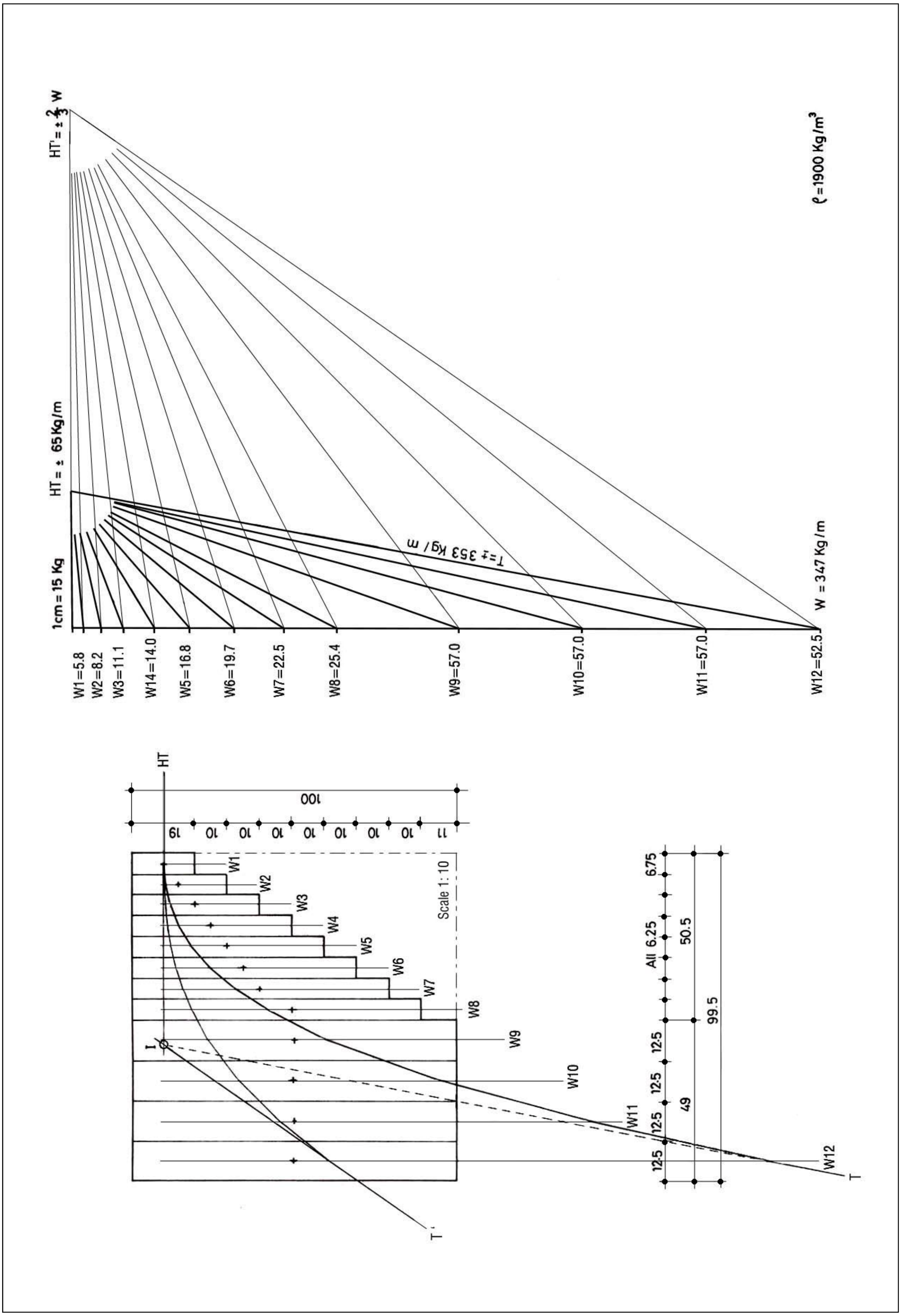


Fig. 68 – Corbelled arch, 101 cm span, 81 cm rise, 24 cm wide, pier = 49 cm

This arch is stable: LT remains within the arch.

2.6 OPTIMISATION STUDIES OF TYPICAL ARCHES

The following optimisation studies show how typical arches can be made stable by decreasing the thickness as the arch rises.

- The chain study first shows the various small chains (or weights) which are required to bring the line of thrust within the middle third of the initial thickness of the arch, as defined at the beginning of the study.
- The funicular study determines, after several adjustments of the thickness, the magnitude of the thrust and the location of the line of thrust within the middle third.
- Once arch stability is achieved, the masonry pattern is studied to build a vault with a combination of horizontal and vertical courses.

Note that the various arches shown in the following examples have an initial thickness of 20 cm at the beginning of the study.

After the optimisation process, the thicknesses are modified as such:

- Semicircular arch 350 cm span : 36.5 cm thickness at the bottom; 7 cm thickness at the top.
- Egyptian arch 360 cm span : 29 cm thickness at the bottom; 7 cm thickness at the top.
- Equilateral arch 360 cm span : 34 cm thickness at the bottom; 7 cm thickness at the top, with a heavy key stone load.
- Bucket arch 346 cm span : Case 1:
  Smooth curve from springer to apex:
  58 cm thickness at the bottom; 7 cm thickness at the top.

  Case 2:
  54 cm wide pier, built like a corbelled arch, with a smooth curve afterwards:
  21 cm thickness against the pier; 7 cm thickness at the top.

1. SEMICIRCULAR ARCH

Funicular study

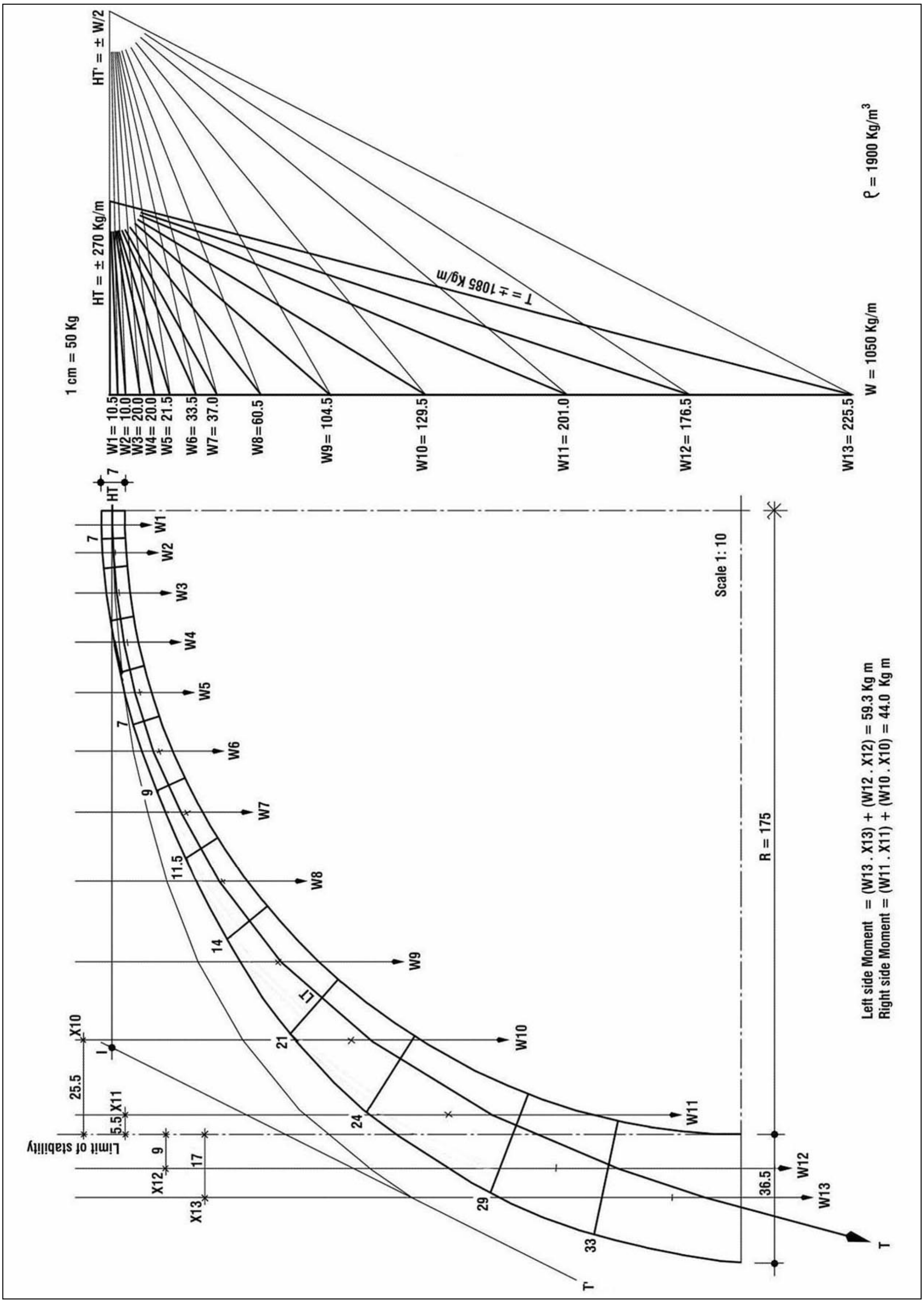


Fig. 69 – Funicular study of an optimised semicircular arch, 350 cm span

Masonry pattern

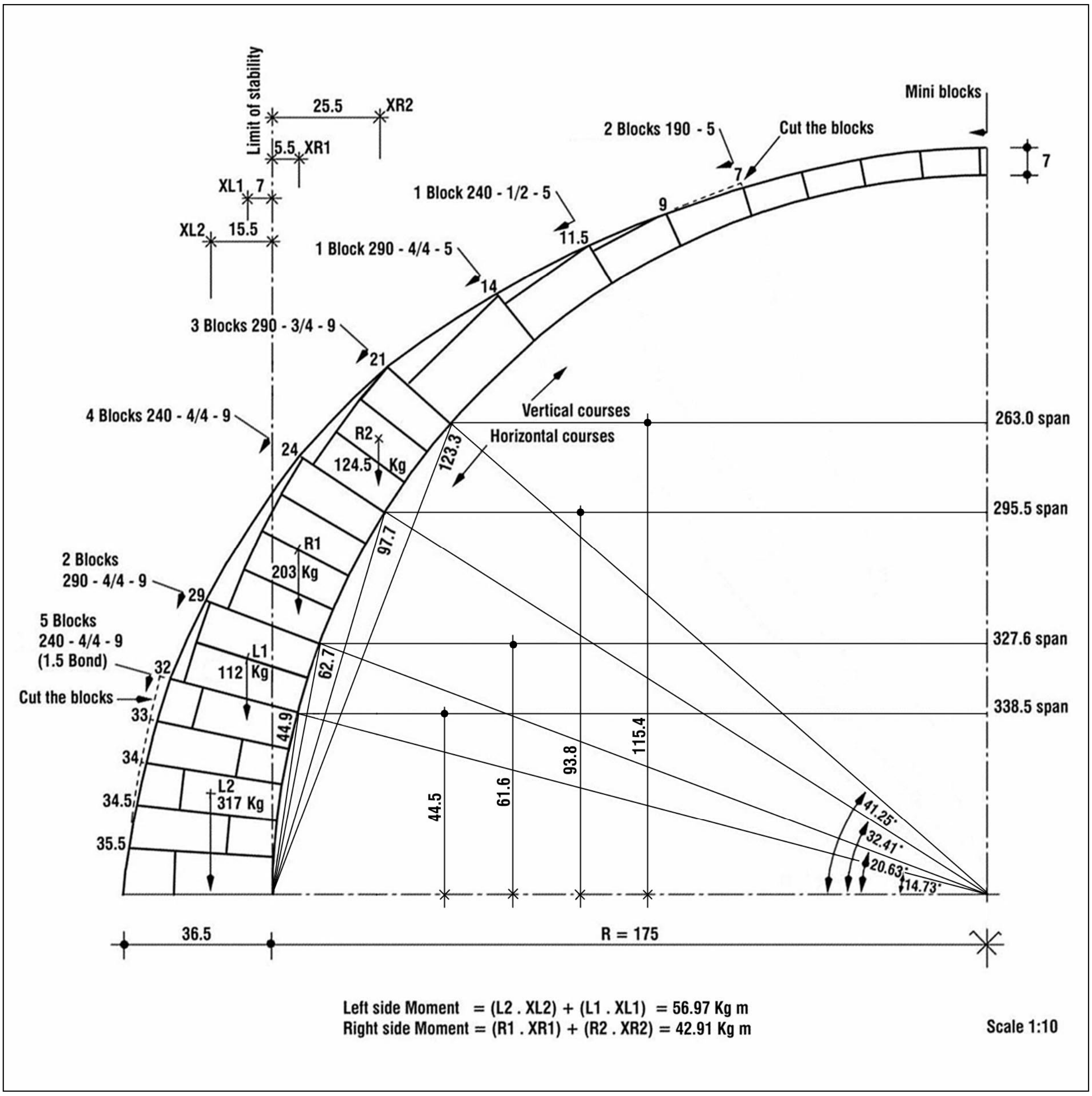


Fig. 70 – Masonry pattern of an optimised semicircular arch, 350 cm span

Label all information required to execute the work on site.

2. EGYPTIAN ARCH

Funicular study

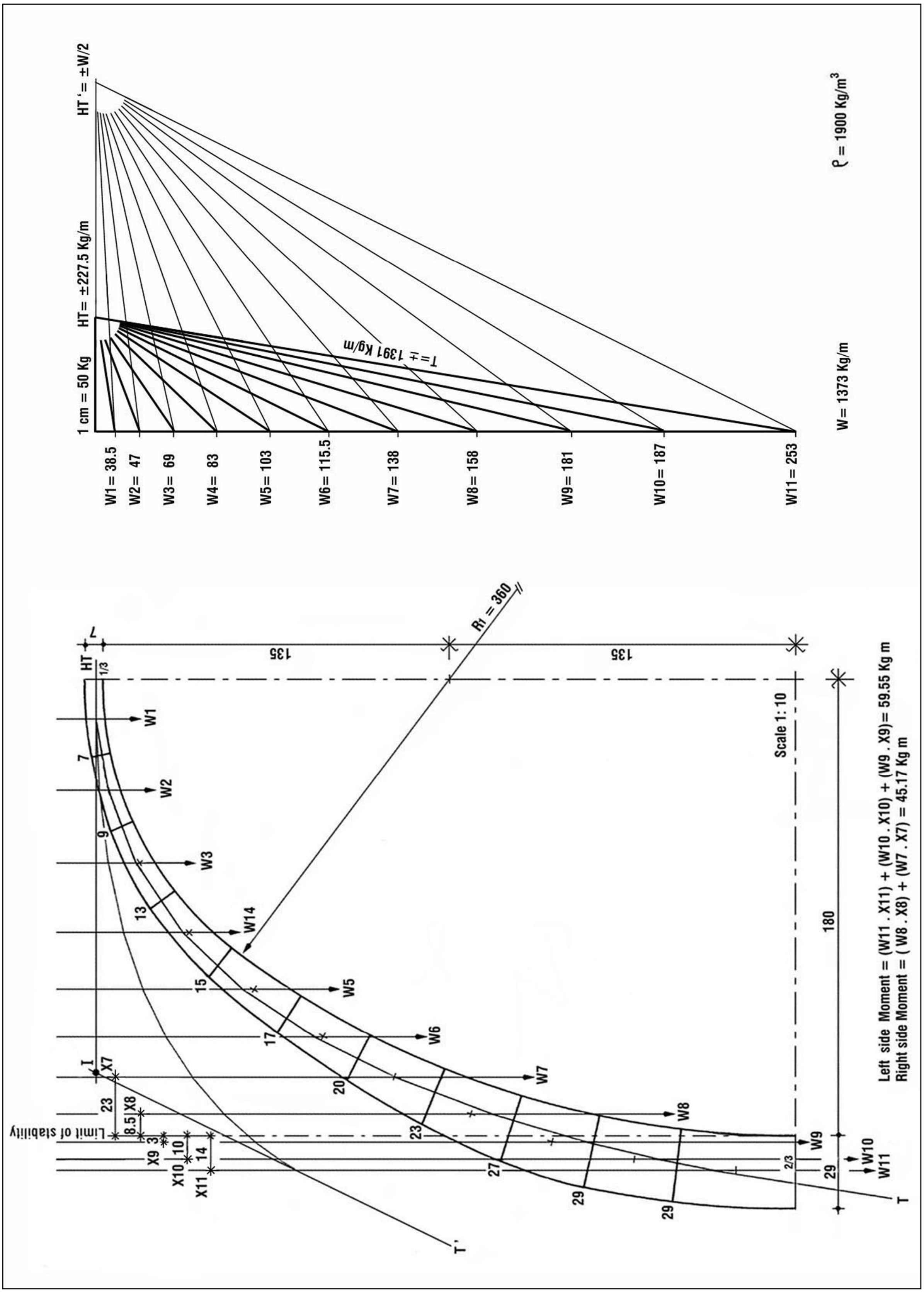


Fig. 71 – Funicular study of an optimised Egyptian arch, 360 cm span

Masonry pattern

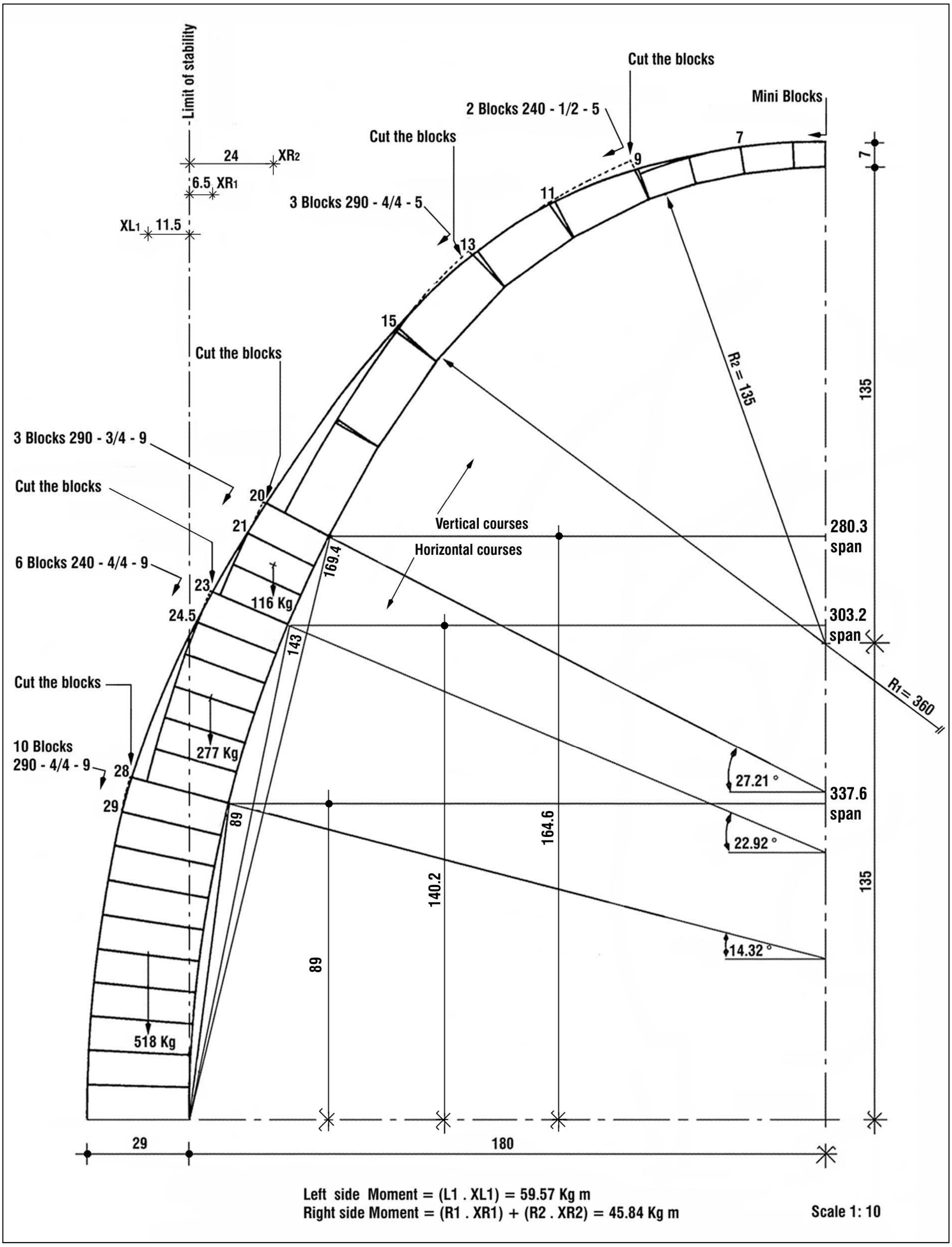


Fig. 72 – Masonry pattern of an optimised Egyptian arch, 360 cm span

Label all information required to execute the work on site.

3. EQUILATERAL ARCH

Funicular study

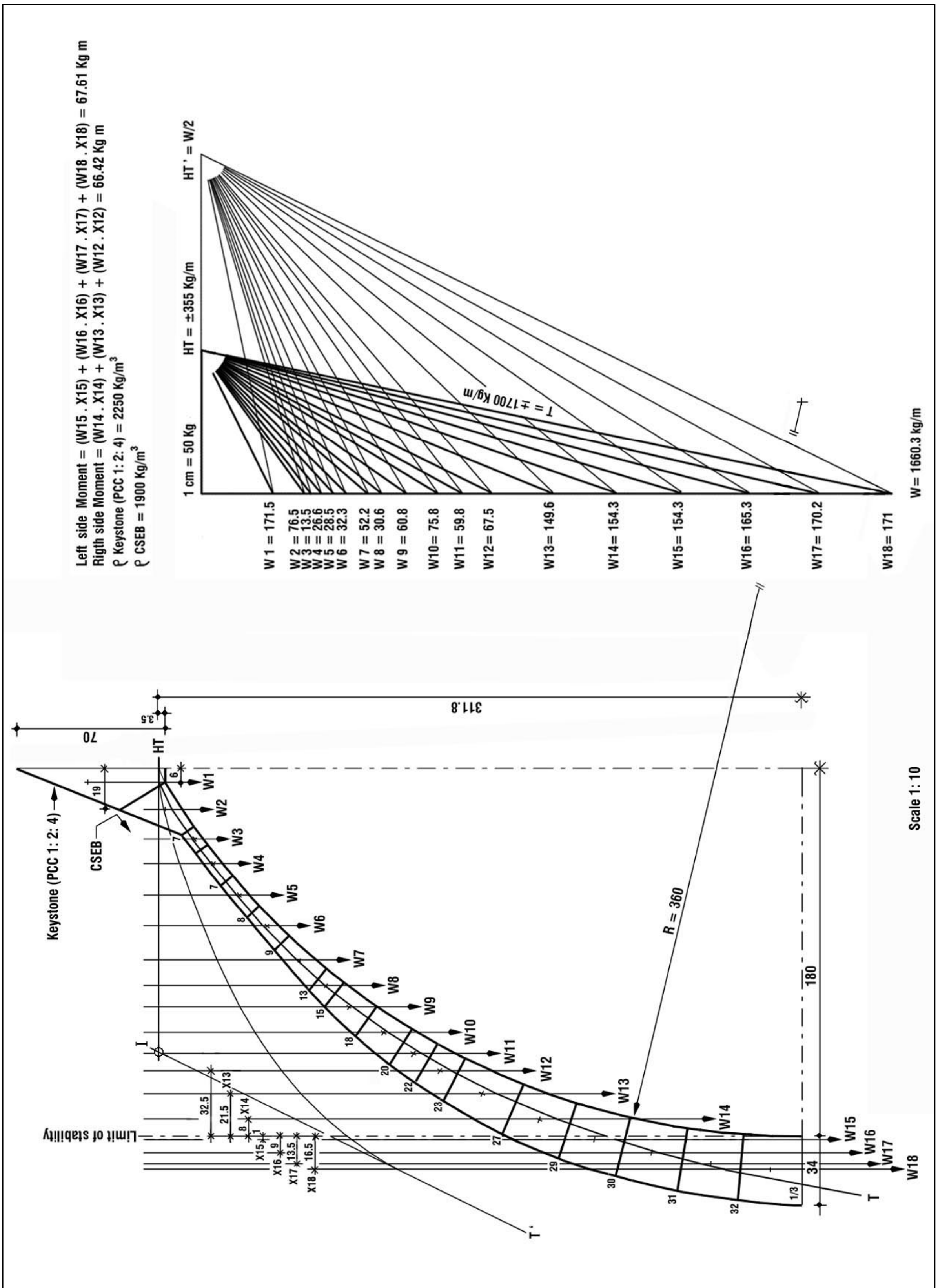


Fig. 73 – Funicular study of an optimised equilateral arch, 360 cm span

Masonry pattern

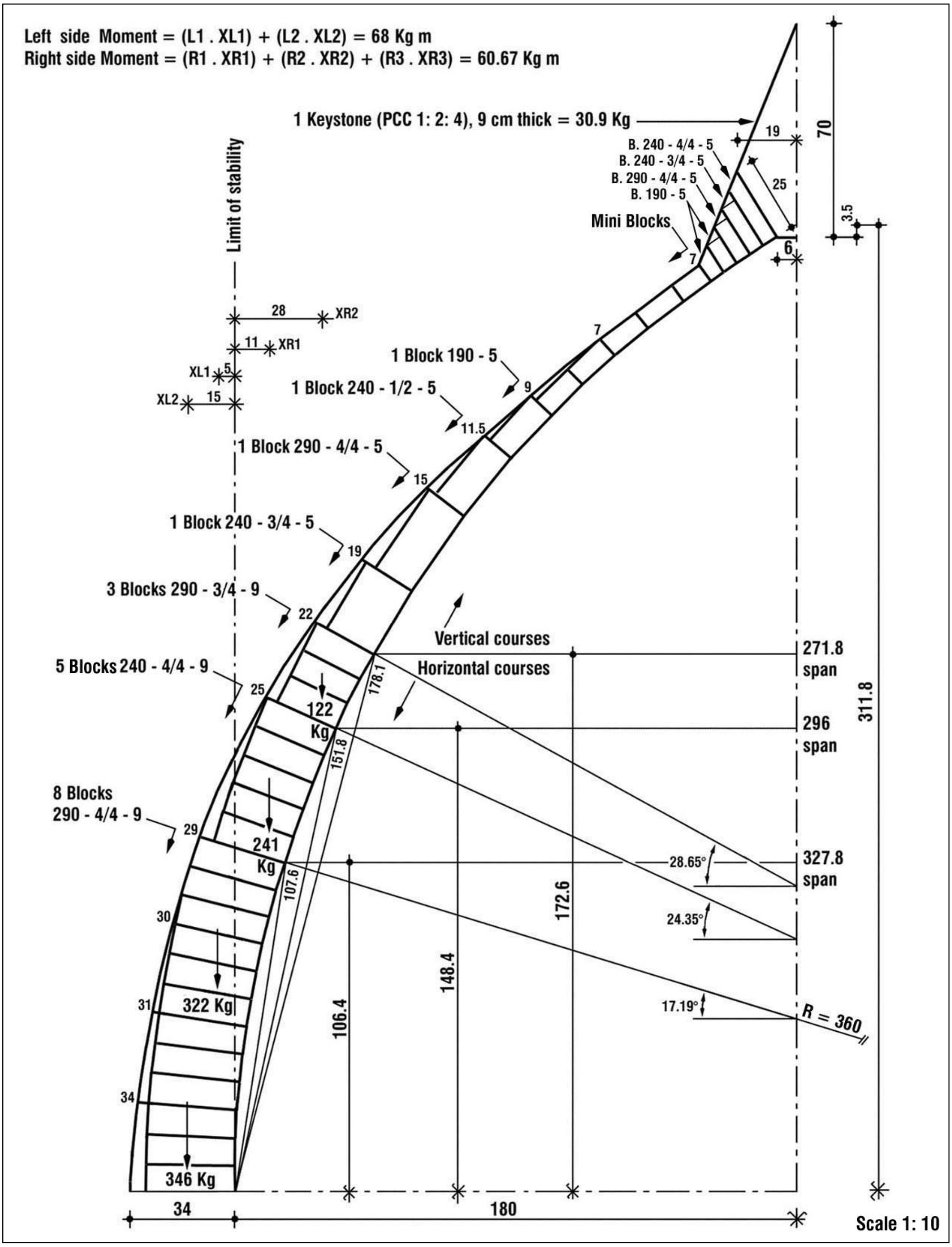


Fig. 74 – Masonry pattern of an optimised equilateral arch, 360 cm span

Label all information required to execute the work on site.

4. BUCKET ARCH

Funicular study (case 1)

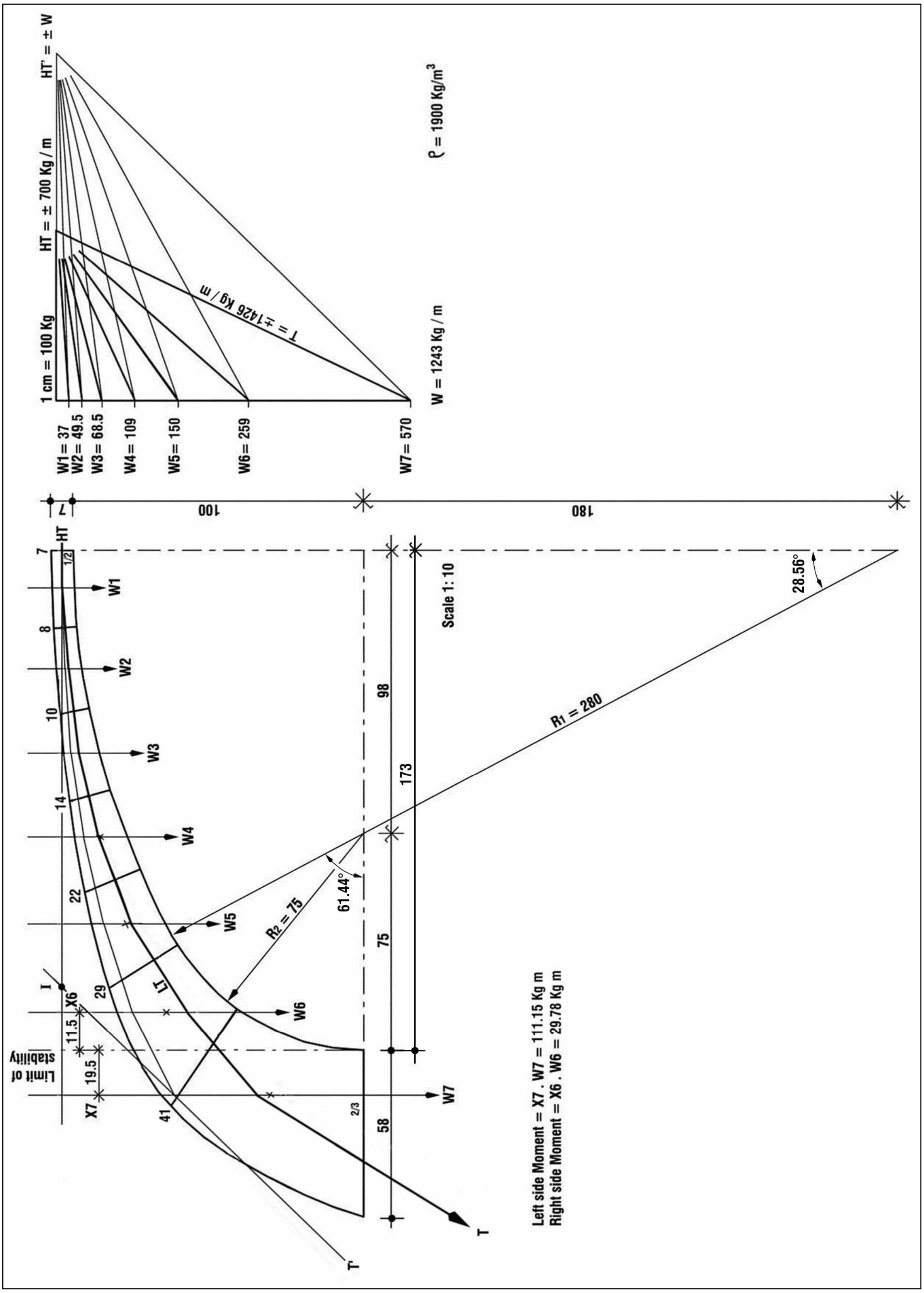


Fig. 75 – Funicular study of an optimised bucket arch, 346 cm span

Funicular study (case 2)

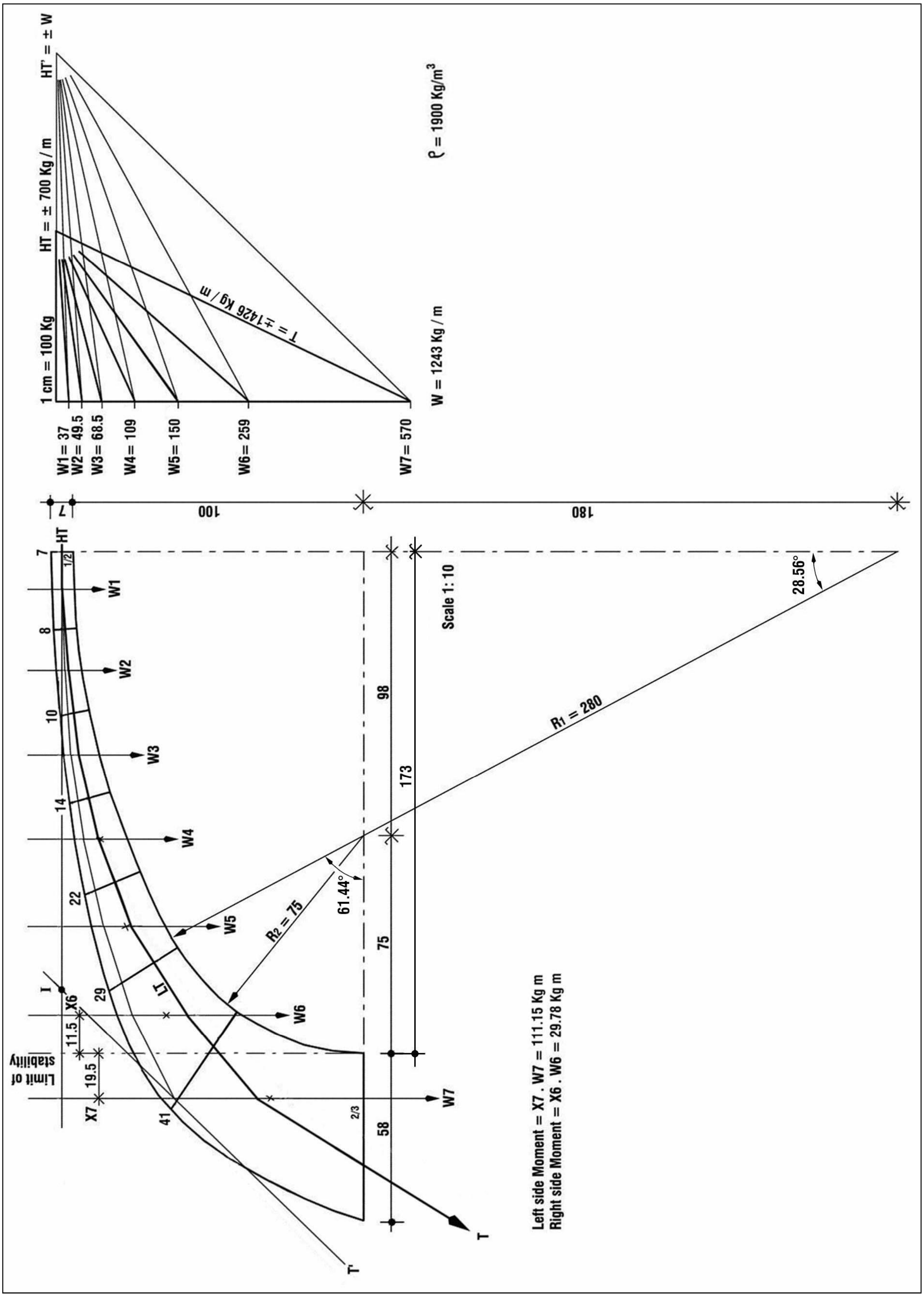


Fig. 76 – Funicular study of an optimised bucket arch, 346 cm span

Masonry pattern (case 1)

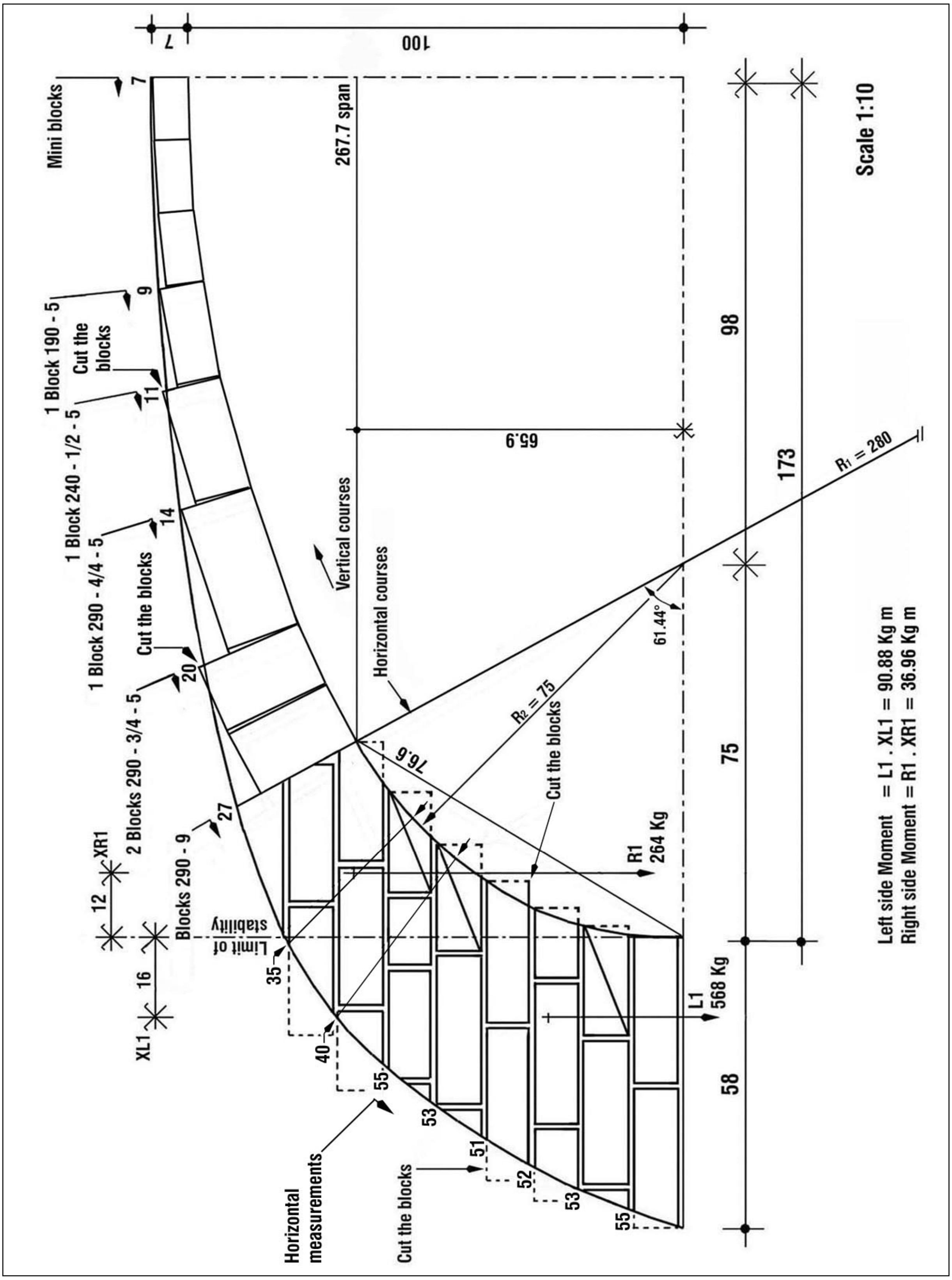


Fig. 77 – Masonry pattern of an optimised bucket arch, 346 cm span (case 1)

Label all information required to execute the work on site.

Masonry pattern (case 2)

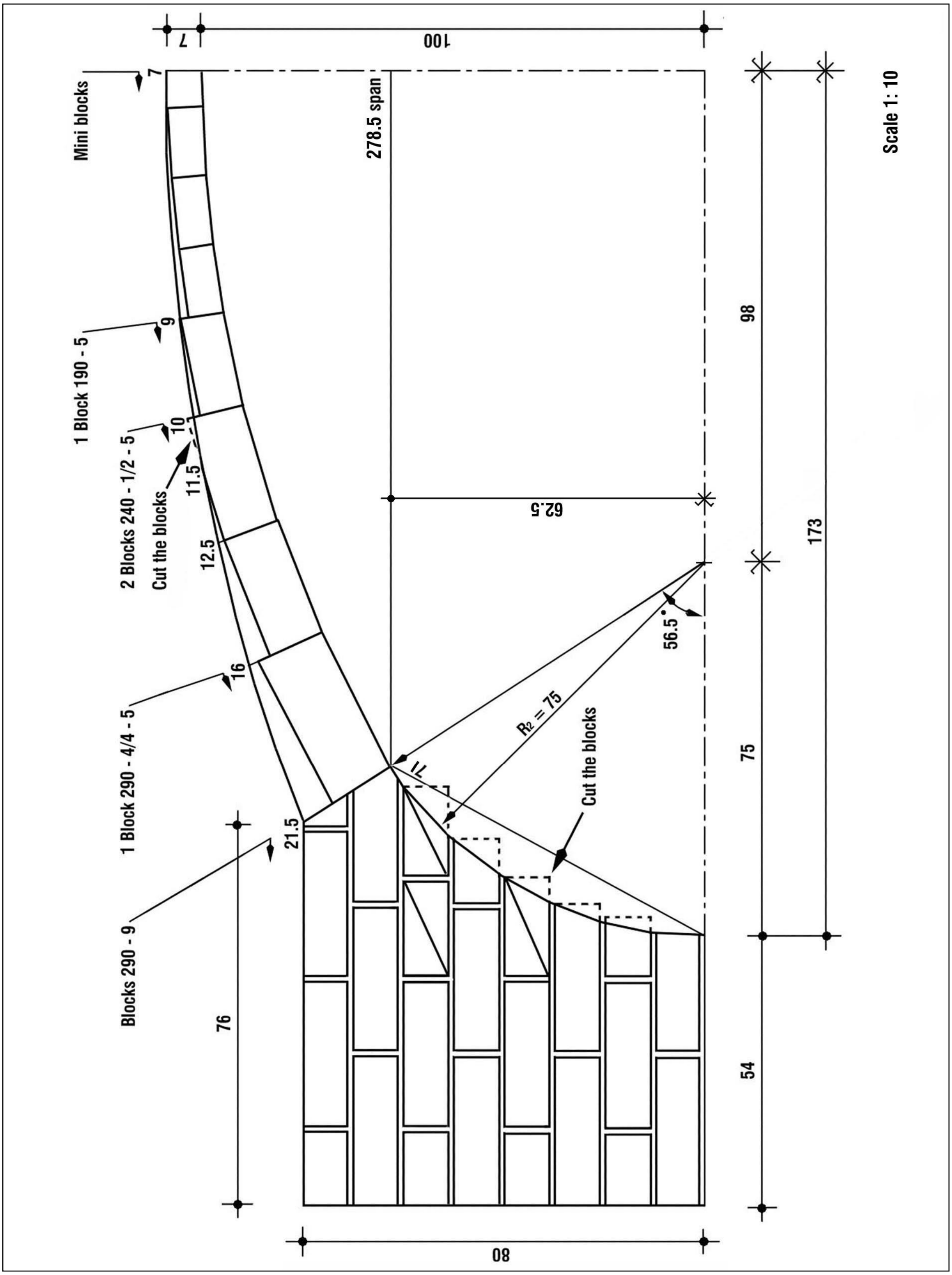


Fig. 78 – Masonry pattern of an optimised bucket arch, 346 cm span (case 2)

Label all information required to execute the work on site.

2.7 EQUILIBRATION OF THRUST FOR ARCHES & VAULTS

Once the geometry and forces have been defined, and it has been determined through graphical analysis that the arch is stable, it is necessary to design the elements for the equilibration (or counter-balance) of the thrust of the arch. The simplest and most effective ways to counter-balance thrust is with the mass of masonry, with a concrete ring beams or with steel tension ties.

2.7.1 Arches

The thrust of arches can often be balanced by the mass of masonry. Tension ties are required only in the case of very large arches or high horizontal thrust values *(See Section 2.7.2: Equilibration of Thrust, Vaults, p. 72)*.

2.7.1.1 Small arches (within a wall)

Small arches are most often employed within a wall and used to span openings such as windows or doors. In this case, the wall itself acts as a buttress to balance the thrust. It has been demonstrated that the angle and magnitude of the thrust will vary according to the type of arch. Accordingly, the more horizontal an arch is (e.g. a segmental arch), the more stress will be exerted within the wall and the longer a wall section must be to ensure that the thrust is transmitted to the foundation.

Two cases can be considered:

1. An arch is located in, or near the centre of a long wall.

   The weight of the wall masonry will counteract the thrust, and no study needs to be conducted.

   (See Section 2.1.7.3: Arch Within a Wall, p. 24)

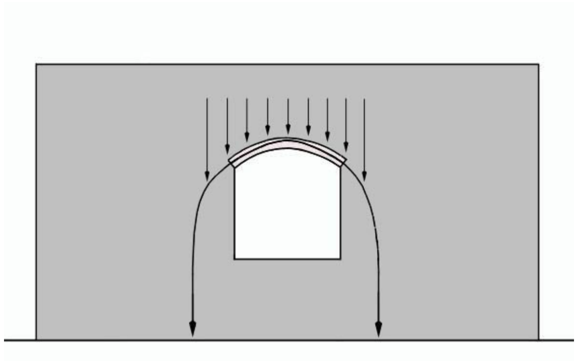


Fig. 79 – Segmental arch centred in a long wall

2. An arch is located near a corner/ end condition of a building.

   Depending upon the geometry of the arch and the exact proximity to the end of the wall, the thrust may compromise the stability of the wall.

   A funicular study must be conducted to check if the weight of the wall masonry is sufficient to counterbalance the thrust.

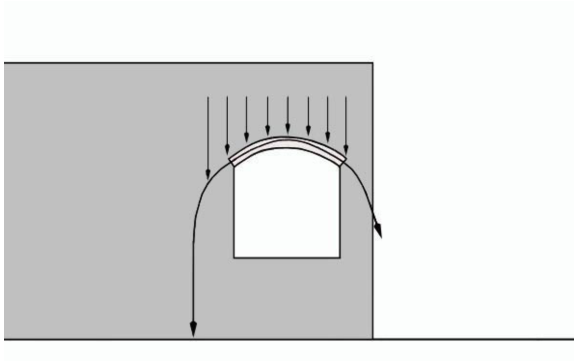


Fig. 80 – Segmental arch in a corner

Fig. 80 shows that the segmental arch is too close to the corner of the wall; this will cause cracking. The line of thrust exits the wall, and there is no compression solution that transfers load safely to the ground.

Fig. 81and Fig. 82 demonstrate two alternative solutions to balance the thrust. In both cases the thrust should be contained within the middle third of the corner pier.

1. Modify the shape of the arch, to achieve a more vertical line of thrust, which can be balanced by the weight of the masonry *(Fig. 81)*.
2. Move the opening away from the corner *(Fig. 82)*.

In both cases, the thrust should be contained within the middle third of the corner pier.

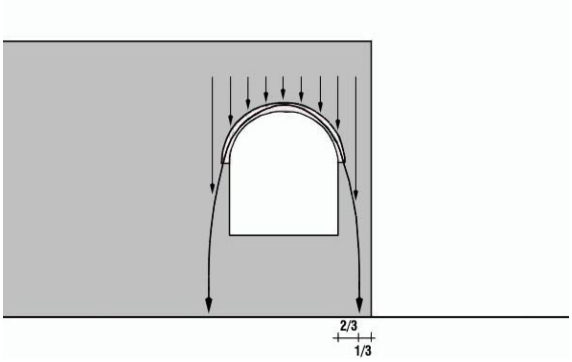


Fig. 81 – Modified shape of the arch in the corner

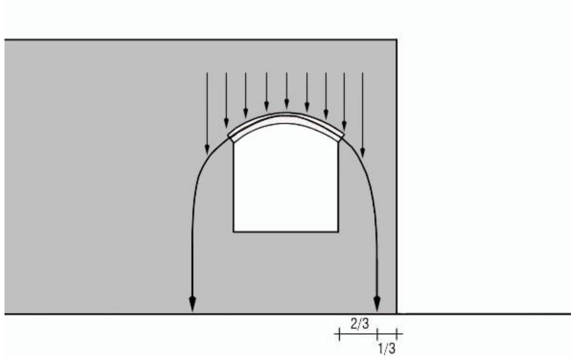


Fig. 82 – Arch moved away from the corner

Another possibility would be to increase the height of the building at the corner, to overload the masonry and redirect the thrust more vertically. This method is often not practical on account of the goals of architectural design and overall harmony of the building. However, for example, this technique was regularly used in European cathedral masonry with the addition of pinnacles and sculptures, where pier overloading was both sculptural and structural.

2.7.1.2 Large arches

Arches of larger spans less commonly have masonry mass to counter-balance thrust. Unreinforced masonry piers should be dimensioned with sufficient width. Stability principles should be respected and a funicular study conducted to check the equilibrium of the structure *(See Section 2.1.4: Principles of Stability, p. 18)*.

Fig. 83 shows that the pier is not wide enough and the arch will collapse, because the line of thrust at the base of the pier. Fig. 84 and Fig. 85 demonstrate two alternative solutions to balance the thrust:

1. Modify the angle of the roof to increase the load of the haunches *(Fig. 84)*.
2. **Maintain the same roof angle, but add a buttress or widen the pier *(Fig. 85)*.**

In both cases, LT should remain within the middle third of the pier.

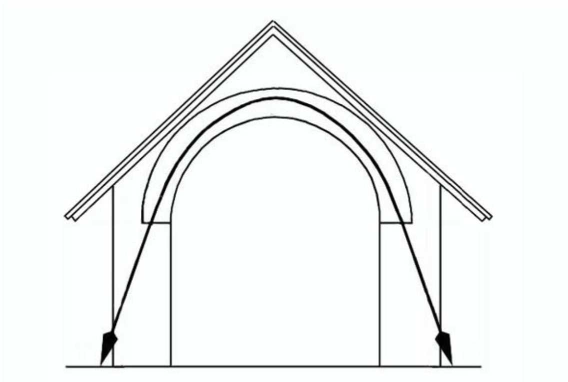


Fig. 83 – Pier not wide enough for a large arch

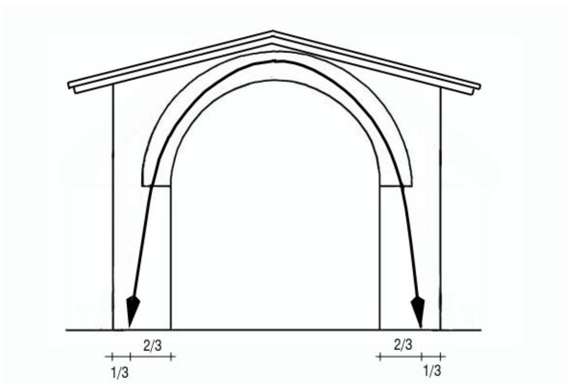


Fig. 84 – Modified angle of the roof for a large arch

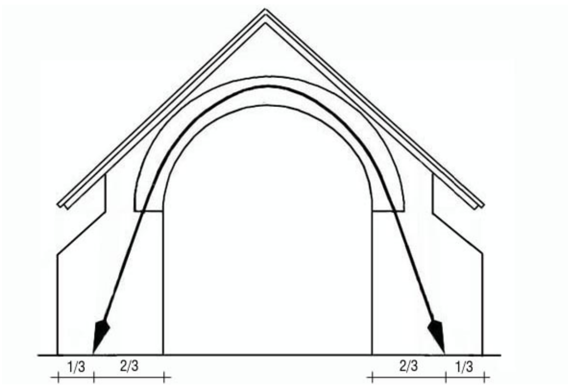


Fig. 85 – Buttress addition for a large arch

Note that the force considered for dimensioning the width of the abutment is HT.

2.7.2 Vaults

The thrust of vaults can be balanced by means of piers, buttresses and tension ties. Stability principles should be respected *(See Section 2.1.4: Principles of Stability, p. 18)*.

If the thrust is to be balanced only by means of abutments, the horizontal thrust component force must be considered for the calculation of the abutment width. The example case here is a vault resting on a beam or ring beam, which tied back by a tension tie.

Beams and ring beams are generally cast with reinforced cement concrete (RCC). Tension ties are generally composed of mild steel. Once the stability study has been completed and the internal forces of the arch are known, the tension tie and beam or ring beam can be calculated using conventional methods.

2.7.2.1 Calculation of a tension tie

Tension ties which are integrated into springer beams can provide a reaction force which “ties back” the outward thrust of a vault.

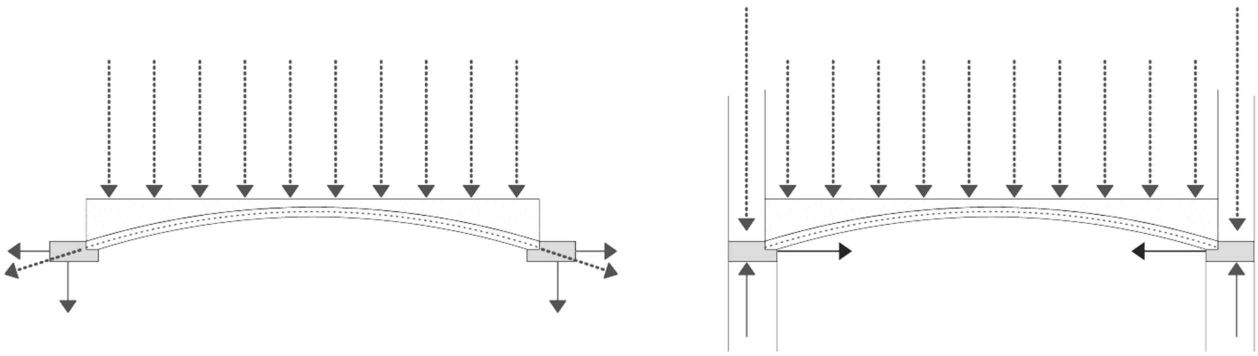


Fig. 86 – Tension tie with reaction force required to counter-balance horizontal thrust

The force F (kg) applied on the tension tie can be calculated as:

F = 2 HT (kg/m) x tension tie spacing (m) Where: 2 is a safety factor and HT the Horizontal thrust.

Round profile rods are most commonly used for tension ties; however, any profile can be used, as long as the section is designed to safely withstand the force. The section of the steel profile should be defined according to the admissible stress of the local steel:

$$\text{Steel section} = \left(\frac{F}{\sigma_{adm.}}\right)\left(cm^2\right)$$

Where: F = Force applied on the tension tie (kg)
$\sigma_{adm.}$ = Admissible stress of steel (kg/cm^2)

The admissible stress for mild steel can be safely considered as 2400 kg/cm^2 (in India). Accordingly, the closest, larger steel section should be chosen.

Example:
HT = 1220 kg/m Tension tie spacing = 1.80 m

⇒ The force applied to the tension tie is: F = 2 x 1220 kg/m x 1.80 m = 4392 kg
⇒ The minimum area of steel required is: 4392 kg / 2400 kg/cm^2 = 1.83 cm^2
⇒ A safe steel section for the tension tie is: 1 No. round rod ∅ 16mm = 2.01 cm^2

2.7.2.2 Tension Tie Connection Detail

Care should be taken to properly anchor the tension tie into the beam or ring beam.

The length of the rod embedment into the RCC beam is essential, to ensure that tensile force is transmitted to the concrete.

When adequate embedment is not possible, another solution to transmit tensile force is to convert the latter into a compression stress by bending the rods into the RCC beam or ring beam.

Fig. 87 shows such a good-practice connection detail of a tension tie into a springer beam.

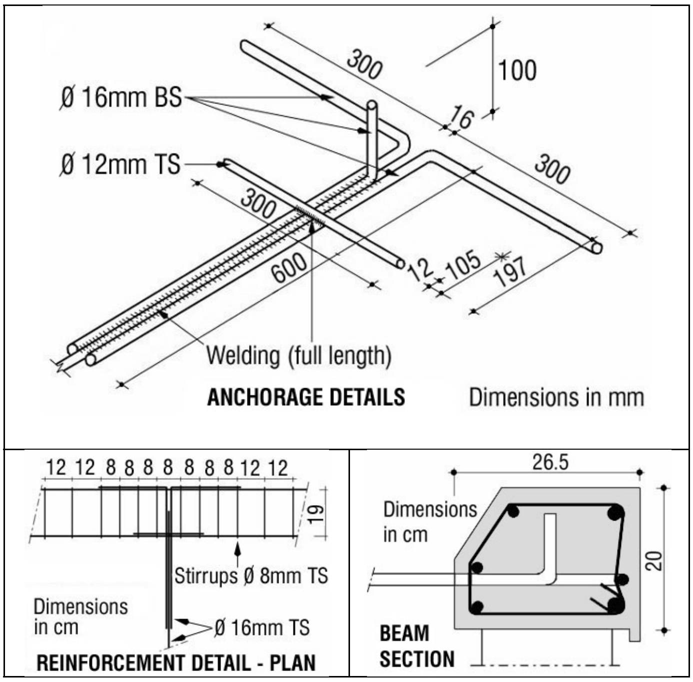


Fig. 87 – Tension tie anchorage by compression

Fig. 88 shows the transmission of the tensile force by a combination of compression and embedment of a tension tie into an RCC ring beam.

In this case, the RCC ring beam was precast with a provision for inserting the tension tie (a ∅ 1-1/4" hole, cast with a PVC pipe which was subsequently removed). Following the casting of the concrete, the tension tie is inserted, then bolted with a plate to distribute the compression force to the ring beam. A slurry of expansive concrete grout is then poured into the casting hole. Finally, the compression plate and the nut are cast over.

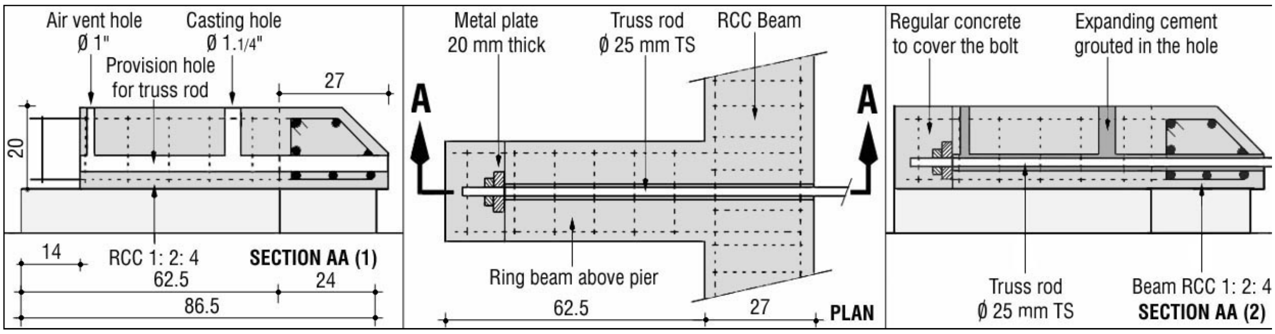


Fig. 88 – Tension tie anchorage by compression and embedment

2.7.2.3 Forces acting on a ring beam

In the case that a vault springs from a ring beam cast over a solid masonry wall (e.g. a ring beam which does not span any opening), only the horizontal thrust must to be considered for the calculation of the RCC element.

The span which must be considered to resist the thrust is the effective spacing between the tension ties.

Thus, the stress exerted on the ring beam is:

Horizontal = HT (kg/m) x tension tie spacing (m)

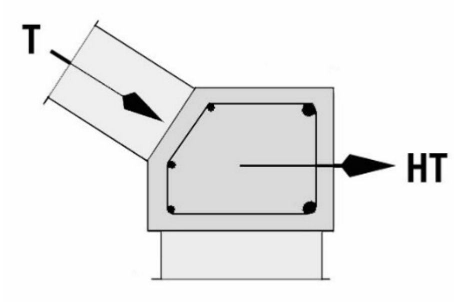


Fig. 89 – Force applied on a ring beam

2.7.2.4 Forces acting on a beam

For the case in which a vault springs from a beam which is spanning an opening (e.g. a transverse beam over a masonry wall, through which there is an opening for a door or a window), the beam must be calculated to withstand both the horizontal and vertical components of the thrust (e.g. the horizontal thrust and self-weight of one half the vault).

The span has to be considered in two directions:

- Vertically, to sustain the load of the vault and span over the opening.
- Horizontally, to span the spacing between the tension ties and sustain the outward thrust of the vault.

Thus, the stress exerted on the beam is:

- Vertical = W (kg/m) x opening span (m)
- Horizontal = HT (kg/m) x tension tie spacing (m)

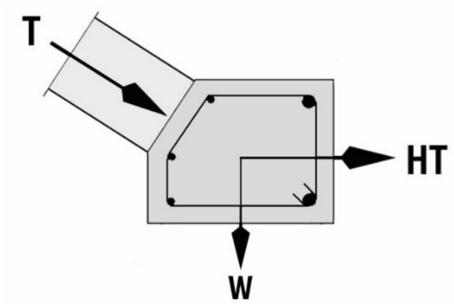


Fig. 90 – Forces applied on a beam

2.7.2.5 Inertia & Bending moment

As the horizontal thrust of the vault can be sometimes quite high, depending upon the ratio of the rise, span and thickness of the vault, the width of the beams should be large enough to achieve an adequate moment of inertia, which can allow for a reduction in the size of the steel rods.

Fig. 91 shows a simple design strategy to increase the inertia with a rainwater gutter cast into the ring beam.

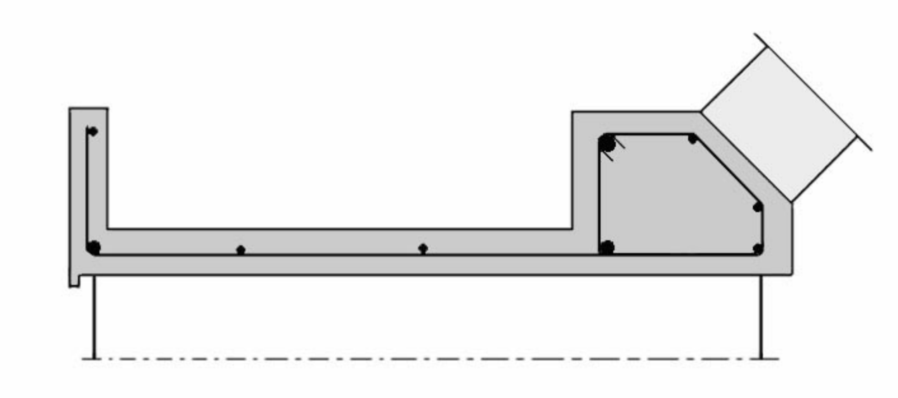


Fig. 91 – Increased inertia of ring beam with a rainwater gutter

Note how the bending moments behave for both cases of beams or ring beams.

Tension ties are typically placed at regular intervals; therefore, the bending moments will be inverted. Thus the bending moments and shear stresses of the beams should be calculated accordingly.

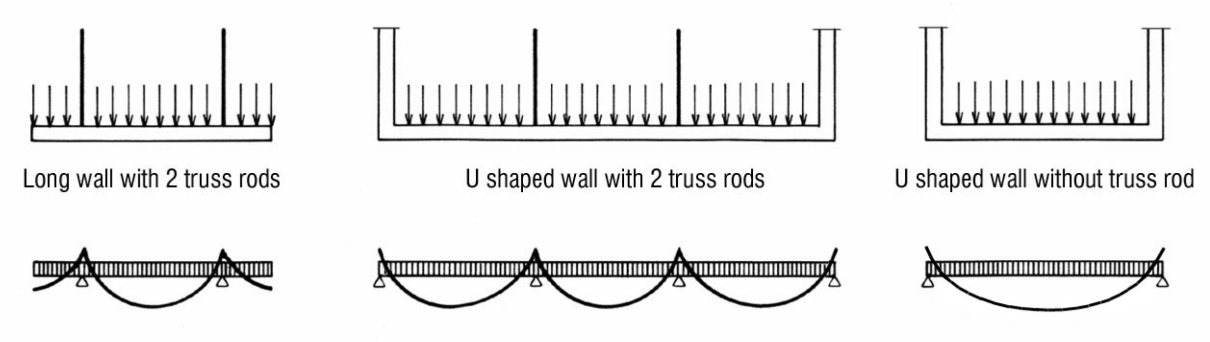
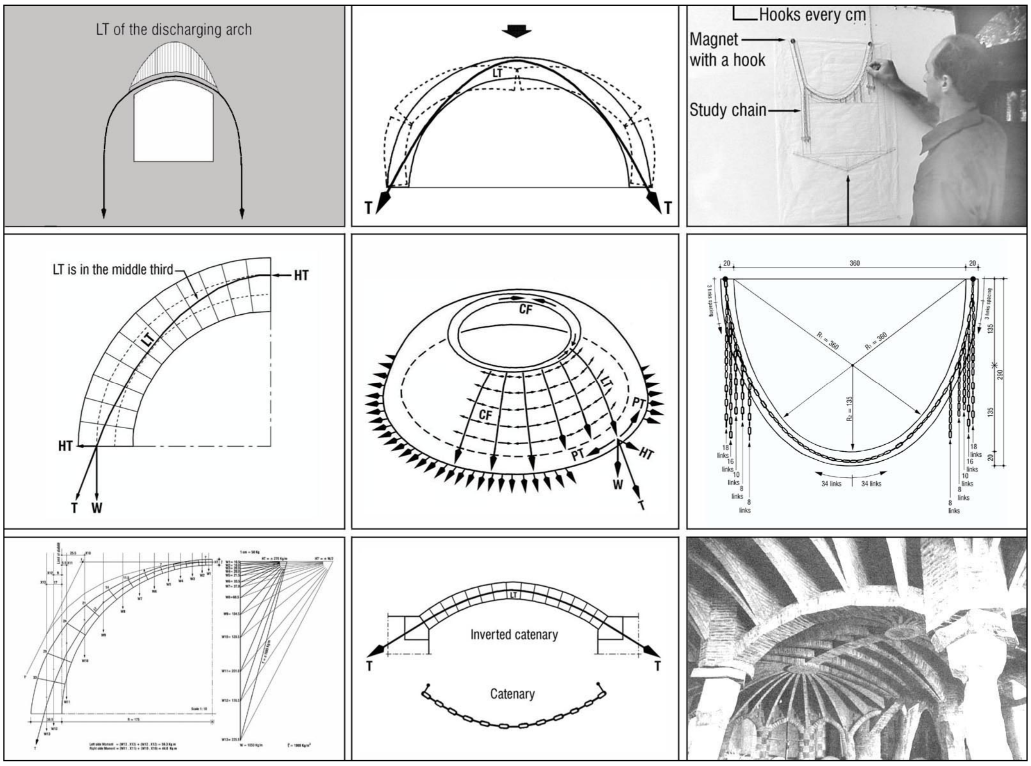


Fig. 92 – Moments acting on beams or ring beams

3. STABILITY OF DOMES



3.1 BASIC STRUCTURAL PRINCIPLES FOR DOMES

3.1.1 Forces Acting in Domes

The forces acting in domes are also a compressive thrust, which determines the stability of the structure. Just as in arches and vaults, the dome's thrust is also the resultant of its weight and the horizontal thrust of the basic arch section. Therefore, there is also a line of thrust in domes (just like that in arches). In domes, however, this line of thrust is called meridian forces.

When a dome is generated by the intersection of two vaults, the forces can be analysed as those of the generating vaults. However, when a dome is created by the rotation of an arch around a vertical axis, other forces are acting: hoop forces (HF). Hoop force is the result of forces acting circumferentially in the dome, a circumferential compressive force.

Domes generated by the rotation of an arch are built with successive horizontal courses. Each block of this course behaves like the voussoir of an arch, transferring a thrust (in the plan of the ring) to the next blocks.

The hoop force in a "circular dome" acts in a horizontal plan (a ring), and can be considered as similar to the thrust which acts downwards in a vertical plan, in the case of arches or vaults. This force explains why it is possible to build circular domes without support. The dome is self-supporting at every stage of its construction, because of the presence of various compression rings. The force of gravity vertically transfers hoop forces into the line of thrust.

HF = Hoop force in every ring
LT = Line of thrust of "an arch" of the dome
HT = Horizontal thrust of "an arch" of the dome
W = Vertical weight of "an arch" and the overload
T = Thrust, resultant force of the horizontal thrust and weight of "an arch"
PT = Peripheral tension (or global tensile hoop force) is created by the combination of the horizontal thrust of all the meridian forces, which radiate from the centre

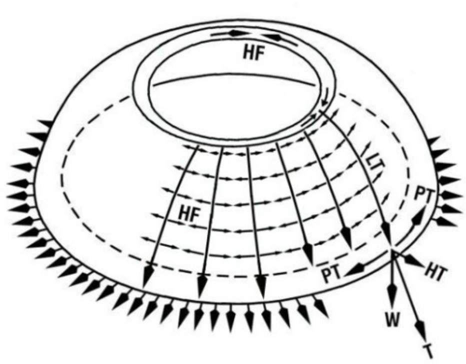


Fig. 93 – Forces in domes

Just as the arch section generating the circular dome rotates around a vertical axis, the dome can be considered as an infinite number of arches whose thrust radiate from the crown towards the base. On the springer level, the accumulation of all these horizontal thrusts will create a peripheral tension (PT), which can tend to push the base of the dome outwards and cause it to crack radially at the base.

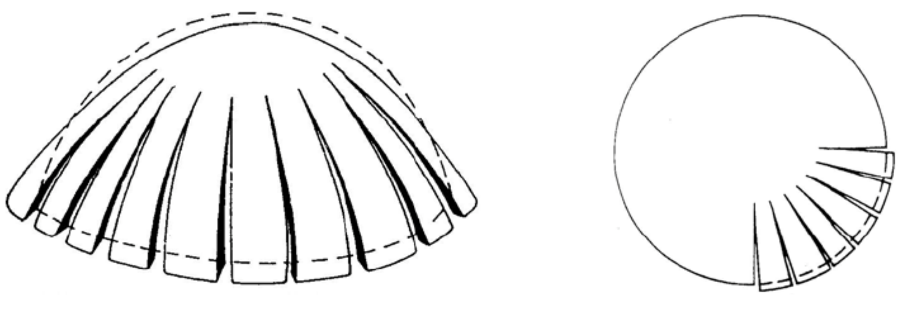


Fig. 94 – Settling behaviour of domes (Heyman 1995)

The combination of the multitude of hoop forces and lines of thrust will create a net of compression forces developed on the entire surface of the dome. Thus, a dome becomes a kind of cohesive nutshell which can resist tremendous stress.

In case of failure of any part of the dome, under an exceptional stress, this net of compressive forces will find another way to act in the dome, and the dome will rarely collapse entirely as long as the supports (walls or columns) are intact.

Note that “circular domes” are generated by concentric circles. They can be spherical, pointed or segmental, and they can be built either on circular or quadrangular plans.

In the case of a quadrangular plan, the intersection of the circular shell and the walls will be:

- A semicircle for a sphere
- A segmental circle for a segmental sphere
- A catenary curve for a pointed dome

The portion of the circular shell in between the walls is called a pendentive.

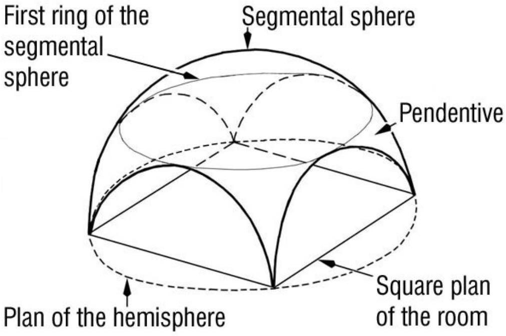


Fig. 95 – Dome on pendentives

3.2 EVALUATION OF THE STABILITY OF DOMES

We have seen that domes which are generated by the intersection of two vaults (i. e. groin and cloister domes), have forces similar to those of vaults. Therefore, the stability of these vaults can be studied like the arch of their cross section or generating geometry. Nevertheless, these kinds of domes have a structural behaviour which is different from that of their generating geometries: they will exert a thrust on four sides, which will require a ring beam or abutments to balance.

For a dome generated by the rotation of an arch around a vertical axis, the hoop forces which act in it cannot be calculated by the methods in this manual. These domes require another approach to calculate their stability.

The examples of domes built all over the world throughout the ages demonstrate that domes can have a wider variety of shapes than vaults. For instance, a dome can be conical with any proportions: either pointed or flatter. But it is obvious that an arch cannot have a triangular section, as a cone is a triangle rotating around a central axis.

Fig. 96 – Conical faceted dome

Fig. 97 – Conical circular dome

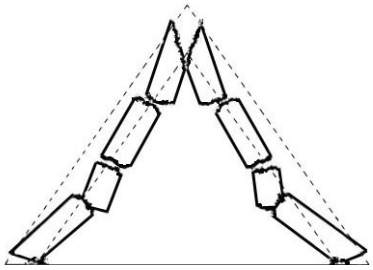


Fig. 98 – Triangular arch

Therefore, if arches or vaults are stable, domes of the same section will necessarily be stable. But the opposite is not necessarily true, as we have seen with the case of the conical dome and the triangular arch.

This gives the principle for studying the stability of circular domes: A cross section of the dome is studied as an arch and; when this arch is stable, the dome will necessarily be stable.

The accumulated horizontal thrusts create a peripheral tension which tends to crack the base of the dome and the support wall. This tension can be evaluated as described in the following pages.

The 22.16 m diameter dome of the Dhyanalinga Temple for Lord Shiva, near Coimbatore-TN-India, was studied with this approach. The dome was built in 9 weeks without any difficulty concerning its stability. The dome has stood since January 1999.

The dome was built on the slope of a hill, and the foundations settled due to the enormous load of the structure: about 1,500 tons, which were built in less than 6 months.

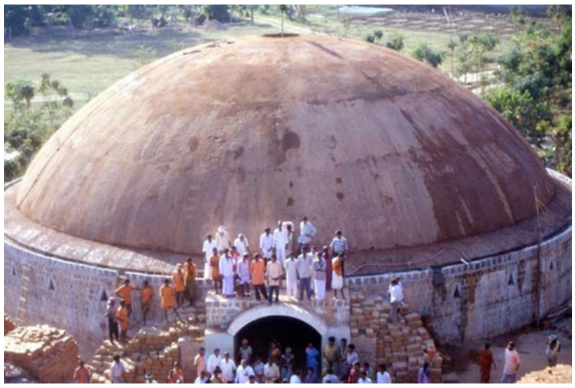


Fig. 99 – Dhyanalinga Temple

Three months after completion, the ground – a black clay – settled unevenly, and one third of the perimeter of the foundations and wall moved a few millimetres outwards and downwards. Thus, the dome cracked above this settlement.

No reinforced concrete ring beam had been used for this dome, as it was a requirement from Swamy Jaggadish Vasudev, the Guru, to achieve a structure with a 1,000-year lifespan. The mass of the masonry wall was studied to neutralize the thrust.

3.3 EQUILIBRATION OF THRUST FOR DOMES

3.3.1 Square Domes

Square domes are generated by the intersection of vaults. Therefore, the forces involved are similar to those of the vaults generating the dome.

The four sides of the ring beam will be subjected to a combination of bending moment and tensile stress, which is equivalent to the thrust resisted by the tension tie.

If tension ties are to be used, their details should be studied following the same principle as for vaults. The ring beam calculations (inertia bending moments and shear stress) are made similarly to the vaults. Note that as the dome is square, care should be taken at the junction of the ring beams, particularly for the anchorage of the rods in the corner.

3.3.2 Circular Domes

We have seen that, geometrically, a dome is the section of an arch which rotates about a vertical axis; and therefore can be considered as an infinite number of arch sections that radiate from the centre of the dome. The thrust of an arch is called Meridian forces in domes.

It is possible to evaluate with the following method the peripheral tension, which tends to open the wall supporting the dome, and to determine the size of the circular tension tie.

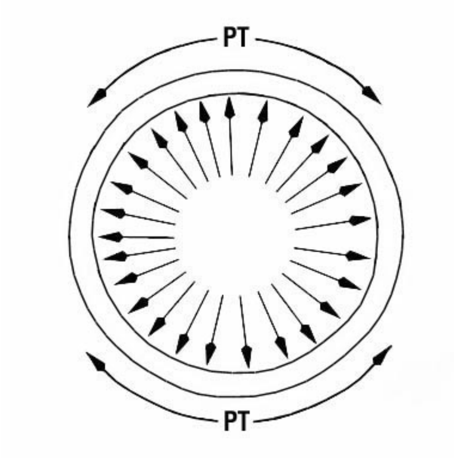


Fig. 100 – Meridian forces of the theoretical arches (lunes)

The forces acting in the theoretical arch section should be evaluated with the optimisation method *(See Section 2.4: Optimisation Method, p. 37)*. The values for the horizontal thrust (HT), the weight (W) and the thrust (T) will be determined as well as the angle of the thrust on the springer.

The dome area has to be calculated *(See Annex 5.2: Geometric Formulas, p. 111)*.
The peripheral tension PT (in kg) and the total weight TW (in kg) of the dome are defined with these formulas:

$$PT = 2HT \cdot \left(\frac{\text{Dome area}}{\pi R} \right) \quad \Rightarrow \quad PT = 4HT \cdot R \text{ for hemispherical dome}$$

$$TW = 2W \cdot \left(\frac{\text{Dome area}}{\pi R} \right) \quad \Rightarrow \quad TW = 4W \cdot R \text{ for hemispherical dome}$$

Where:
HT = horizontal thrust of half the theoretical arch, in kg/m
W = weight of half the theoretical arch, in kg/m
R = radius of the theoretical arch and the dome, in m

Example

A hemispherical dome of 350 cm diameter is studied and its inner area is: $2\pi R^2$

The optimal section for the theoretical arch is defined with the optimisation method *(See Example, p. 60)*, where:

R= 1.75 m HT = 270 kg/m W = 1050 kg/m

The admissible stress for mild steel is 2,400 kg/cm^2.

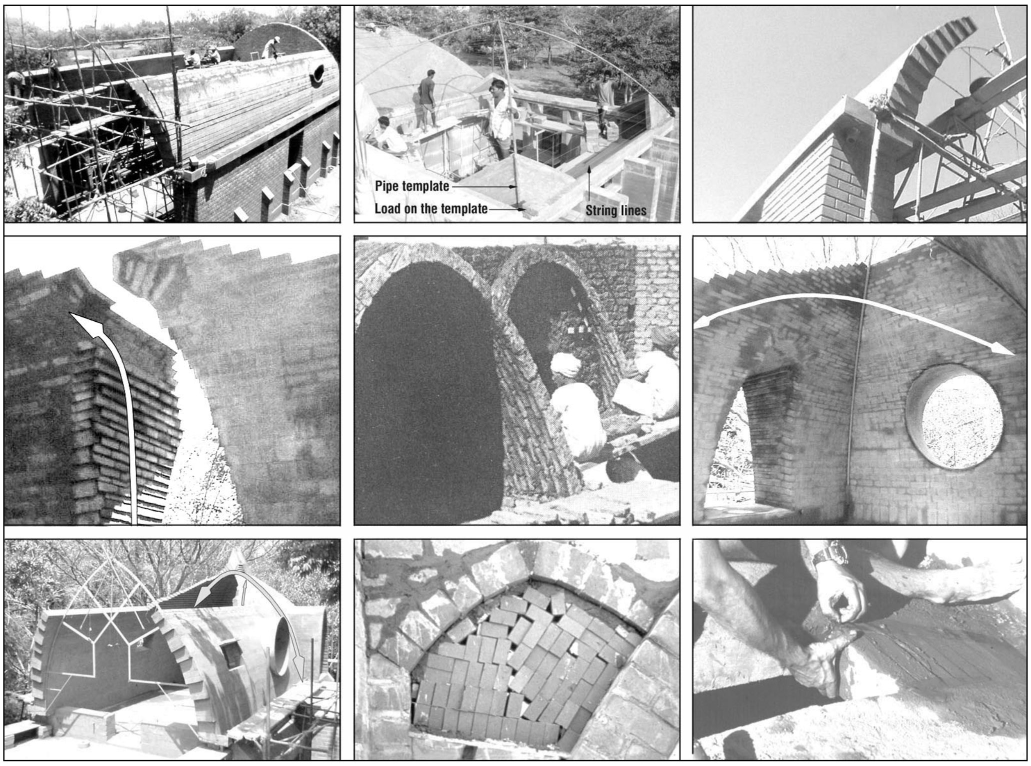
$$\Rightarrow\ PT = 2 \times 270 kg/m \times \left(\frac{(2 \times 1.75 m^2)\pi}{1.75 m \cdot \pi} \right) = 1{,}890 kg$$

$$\Rightarrow\ TW = 2 \times 1050 kg/m \times \left(\frac{(2 \times 1.75 m^2)\pi}{1.75 m \cdot \pi} \right) = 7{,}350 kg$$

⇒ The minimum steel area is: 1890 kg / 2400 kg/cm^2 = 0.7875 cm^2

⇒ A safe steel section for the ring beam is: 2 Nos. Tor steel rod ∅ 8 mm = 1 cm^2

4. CONSTRUCTION OF ARCHES, VAULTS & DOMES



4.1 INTRODUCTORY NOTE

The following chapter presents various specifications and construction details for the construction of arches, vaults and domes (AVD) with Compressed Stabilised Earth Blocks (CSEB).

- ***These blocks must have been well cured for 1 month and left to dry for 3 more months before being used to build AVD structures.*** *The reason for this is that earth blocks, even stabilised ones, shrink on account of the clay in the soil. This time period is essential to allow the blocks to shrink fully. If this requirement is not followed and the blocks are used too early, the shrinkage stress within the vaulted structures can cause cracking in the vault.*

It is essential to understand that cracking is a natural and healthy behaviour in masonry structures. Even structures which are well built can crack. Cracking occurs on account of the material properties of masonry, mainly in response to fluctuations in the environment (Huerta).

Compressed stabilised earth blocks used to build AVD structures should have a very accurate and regular thickness. The Auram Press 3000 allows a tolerance within 0.5 mm for the block thickness, from one block corner to another, and from one block to another. It is essential to regularly control the dimensional thickness during the production process.

The following pages briefly summarize how to build various types of AVD. Nevertheless, one must understand that nothing replaces hands-on experience, or the acquisition of technical skill and coordination which may be gained through a proper training course.

4.2 NUBIAN TECHNIQUE

Fig. 101 – Ramasseum, ~1300 BC

This technique originates from Nubia, in the south of Egypt, hence the name "Nubian". It has been used throughout the ages, as testified by the vaults of the granaries of the Ramasseum at Gourna, Egypt, which were built during the 19th Dynasty, around 1,300 BC.

Fig. 102 – Hassan Fathy

The Nubian technique was revived and disseminated by the Egyptian architect Hassan Fathy (Fathy, 1976). We owe him a debt of gratitude for the global 20th century renaissance of earthen architecture and construction with arches, vaults and domes. CRATerre (The International Centre for Earth Construction) and the Auroville Earth Institute have inherited his spirit and commitment to earth as a building material and its social impact.

The Nubian technique traditionally requires a backup wall on which to stick the blocks. Vaults were built arch-after-arch and the courses were laid slightly leaning. The binder, about 1 to 1.5 cm thick, was the silty-clayey soil from the Nile and the blocks used were adobes (sun dried bricks). The unevenness of the adobes made it necessary to slightly incline the courses, so as to improve adhesion with the force of gravity.

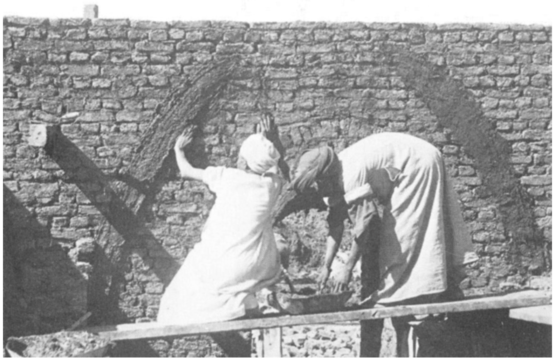


Fig. 103 – Shaping a curve on the adobe wall (Fathy)

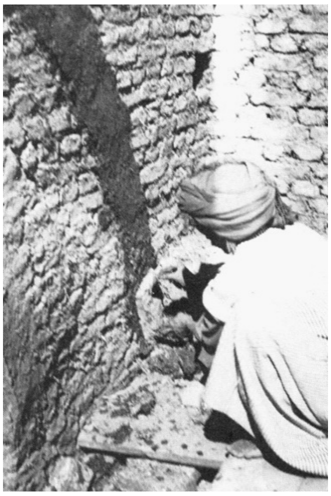
Fig. 104 – Adjusting the curve (Fathy)

The basis of this technique is that the blocks adhere to each other with an earthen glue. In principle, dryer block draws in water by capillary suction and the clay component of the soil acts as an adhesive to bind the blocks. It is essential that the blocks are very thin, to have a high ratio of "surface area to weight"; the larger the surface area and the lighter the block is, the better its adhesion to a surface is.

The Nubian technique was also used for building circular domes with a compass, similarly to the method demonstrated in this chapter. This technique has the advantage of allowing one to build vaults and domes without centring, which is a considerable cost for construction and requires resources which are not always available (e.g. wood, on account of climate or deforestation).

This technique with vertically inclined courses has a major disadvantage. The earth glue is very liquid and the blocks are very thin, therefore the shrinkage of the glue can induce cracks, especially in vaults. When Compressed Stabilised Earth Blocks are used to build vaults using this technique, the course can be absolutely vertical, because it is no longer necessary to incline the courses for adequate adhesion. The regularity of the block insures that the mortar joint can be sufficiently thin for excellent capillary suction and adhesion of the block. The even regularity of CSEB produced by the Auram press 3000 allows building with a cement-stabilised earth glue of only 1-2 mm in thickness.

The Nubian technique has been further developed by the Auroville Earth Institute to build other types of vaults, such as cloister and groin domes, and has been used as the basis of the Earth Institute's "Free Spanning" technique.



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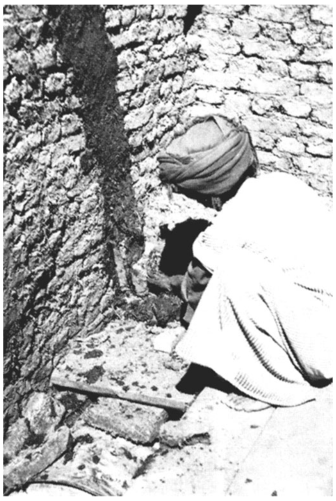

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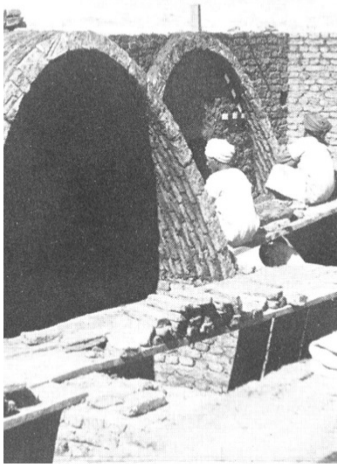


i

Fig. 105 – Nubian vault construction

a. Starting the vault; b. Starting the inclined course; c. First course;
d. Second course; e. Applying some mortar; f. Third course;
g. Fourth course; h. The first arch is complete; i. Building arch after arch.
(Hassan Fathy)

4.3 "FREE SPANNING" TECHNIQUE

The "Free Spanning technique" is a modification of the Nubian technique, which has been developed by the Auroville Earth Institute. This technique allows courses to be laid either with horizontal courses, vertical courses or a combination of horizontal courses and vertical courses, depending on the shape of the vault to be built.

Fig. 106 – The vault rises with horizontal courses

Fig. 107 – Building a semicircular vault of 6 m span

In the classical form of the Free Spanning technique (Fig. 108), the first courses are laid horizontally and then the vault is closed with vertical courses. Like in the Nubian technique, the vertical courses are set with the adhesive behaviour of an earth mortar. The horizontal courses are, however, not stable on account of the adhesion of the blocks by earth mortar, but rather by the equilibrium of gravity forces of the various courses as it is transferred through the masonry. It is essential to study the location of the centres of gravity in the masonry, to ensure that the weight of the masonry with horizontal coursing (considered at the centre of gravity) never extends beyond the limit of the springers.

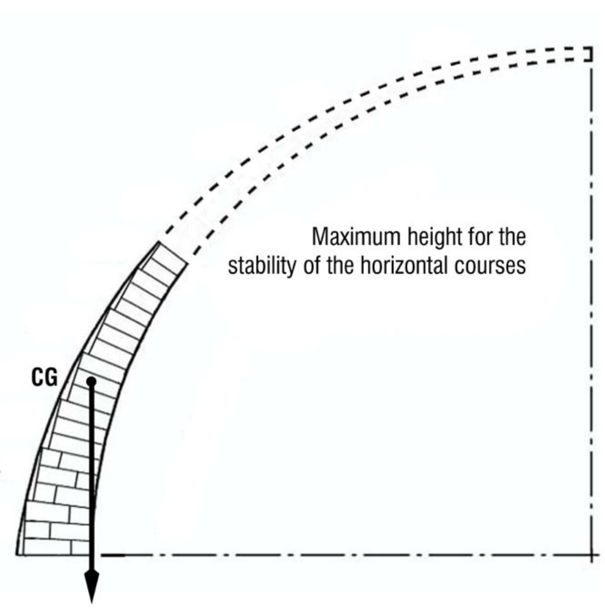


Fig. 108 – Limit of stability of the horizontal courses

Fig. 109 – Load transfer in the shape of a catenary in an equilateral vault with a half dome

The horizontal courses are built in cantilever, always keeping the center of gravity within the section of the footing, so that the cantilevering vault is stable.

The vertical courses are then built with the principles of the Nubian technique, relying on capillary adhesion to close each subsequent course. As soon as one arch is fully closed, that portion of the vault is stable, and then next vertical course may be begun *(Fig. 108)*.

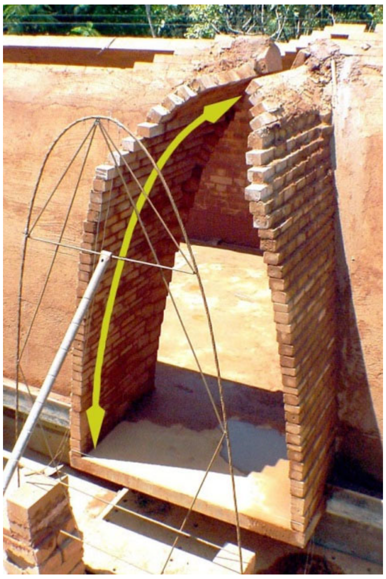


Fig. 110 – Force as a rampant arch

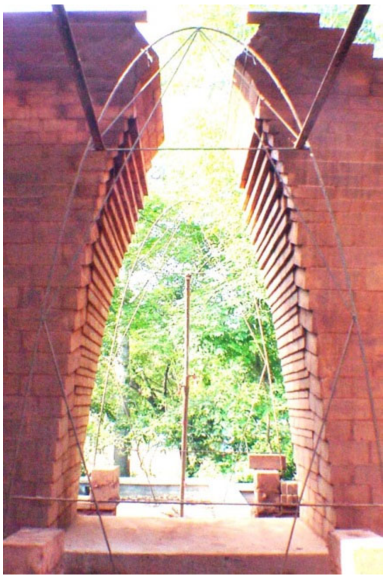


Fig. 111 – Equilibrium of forces

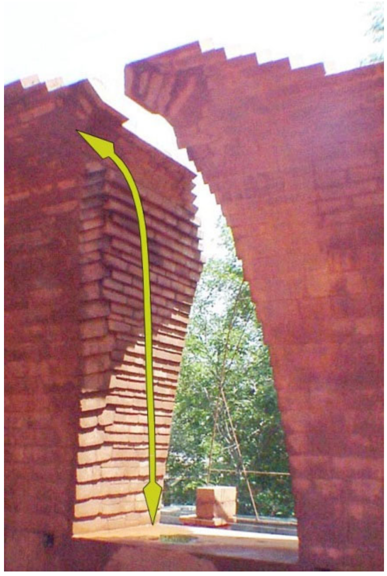


Fig. 112 – Force as a rampant arch

The vault, being built with horizontal courses, rises like a corbel which is curved and has courses inclined at the same angle as the radius of the curve.

Fig. 113 shows the curved corbel of the left side of a 6 m span semicircular vault.

It is here at the maximum height, beyond which the vault would collapse inwards; thus, it is the limit of stability.

Fig. 113 – Limit of stability of the curved corbel

Fig. 114 shows the maximum height of horizontal courses which can be built the full length of the 6 m span semicircular vault.

Four more courses can be built horizontally by steps, as the load transfer can then take the form of a rampant arch within the masonry.

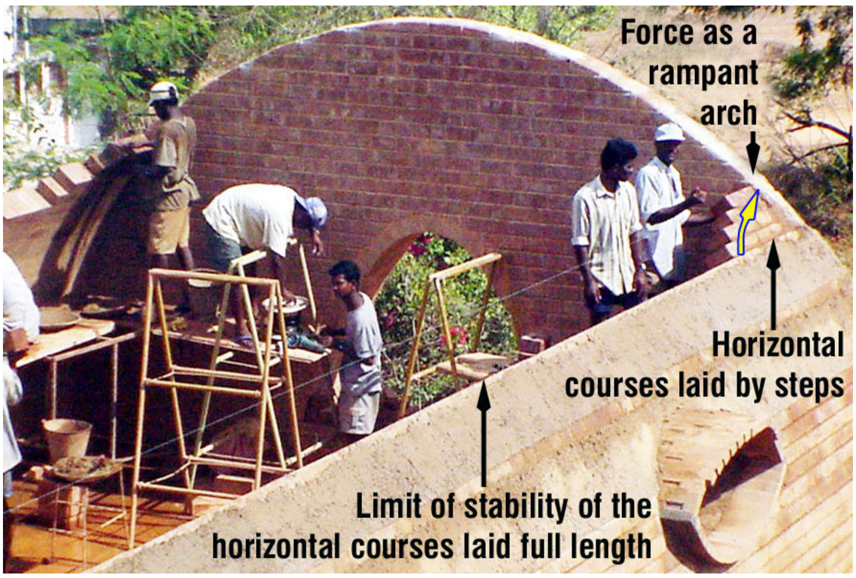


Fig. 114 – Beginning horizontal steps

Fig. 115 shows that the courses are too inclined. The 6 m span semicircular vault cannot be built horizontally any further, the vault will no longer be in equilibrium. The construction has to continue with vertical courses. Fig. 116 shows that the horizontal courses of the 3.60 m span equilateral vault have reached their maximum height. The courses should now be laid by steps. Fig. 117 shows that the masonry continues with horizontal steps, to increase the structural depth for better load transfer into the half dome at the end. Fig. 118 shows that the forces pass through the keystone of the equilateral vault.

Fig. 115 – Beginning vertical courses

Fig. 116 – Equilateral vault with horizontal courses

Fig. 117 – Horizontal courses by steps

Fig. 118 – Forces through the keystone

The Free Spanning technique with horizontal courses presents an advantage compared to the Nubian technique: the mortar is sandier and the quantity of mortar is proportionally less, as the blocks are larger. Therefore, the vault tends to crack less because there is less shrinkage due to the mortar.

Very flat segmental vaults and certain shapes of vaulted structures (i.e. groin vaults) cannot be built with horizontal courses. In these cases, a stable section for the load transfer cannot be found within a cantilevering section of the vault.

The Free Spanning technique demonstrates its full potential with a combination of horizontal and vertical courses. This technique is the most efficient method to build vaulted structures which have been studied with the optimisation method.

4.4 BINDER QUALITY

4.4.1 Soil Identification

The binders are made of stabilised earth. It is essential to identify the soil quality in order to define the binder specifications. Easy soil identification can be done with the preparation of sample tests for the mortar. The specifications for wall masonry should be defined first.

➢ **Three principles for stabilised earth mortar of walls**
- Stabilise 1.5 times more than for CSEB, to achieve the same strength as the CSEB.
- Add coarse sand (0.2 to 2 mm) to reduce the shrinkage when drying.
- Prepare a rather dry, plastic mix: it must not be too wet.

➢ **Conducting tests for stabilised earth mortar of walls**
- Stabilise with 7.5 % of cement, which is approximately: 1 cement: 4 soil: 8 sand (by volume)
- Apply a layer of 1 cm of mortar on a cured block which has been soaked first in water.
- Cure the mortar on the block for 3 days and then let it dry for 3 days (not in direct sun).
- No crack should appear and the mortar should not be crumbly.
- If cracks occur: the soil is too clayey. The soil/sand ratio should be decreased.
  Redo the test with the following mix: 1 cement: 3 soil: 9 sand
  If this sample still cracks, decrease the soil/sand ratio again: 1 cement: 2 soil: 10 sand or further.
- If the mortar is too crumbly, the soil is too sandy: the soil/sand ratio should be increased. Repeat the test with the following mix: 1 cement: 5 soil: 7 sand
  If the sample is still crumbly, increase the soil/sand ratio again to: 1 cement: 6 soil: 6 sand or more.
- Once the mortar for walls is satisfactory, the following specifications can be given for AVD.

For more details about soil identification, see the Auroville Earth Institute publication:
Soil identification for earth construction – Ref. I 10

4.4.2 Arches

The binder for arches is a mortar which should be sandier than that for walls, in order to reduce the shrinkage once it dries. Note that soil and sand should be sieved with a 1 mm mesh.

➢ If the mortar for walls (1 cement: 4 soil: 8 sand) gives satisfactory results, the same mix can successfully be used for arches: 1 cement: 4 soil: 8 sand.

➢ If the mortar for walls is 1 cement: 3 soil: 9 sand, meaning that the soil is too clayey, the specification for arches should be 1 cement: 3 soil: 9 sand or, if needed, less soil and more sand.

➢ If the mortar for walls is 1 cement: 7 soil: 5 sand, meaning that the soil is too sandy, the specification for arches should be 1cement: 9 soil: 3 sand or, if needed, more soil and less sand.

The fluidity of the mortar varies with the type of arch:

➢ Semicircular or pointed arches need a fluid mortar: add more water than the mortar for walls.

➢ Segmental arches need two fluidities for the mortar. The same mortar is prepared in 2 pans:
  - One mortar is very liquid. It is like a glue and is used for the bottom part of the joints.
  - The other mortar is much dryer. It is used to fill very tightly the upper part of the joints. Note that this dryer mortar is used once the arch has been completed and before removing the centring. This mortar is compressed with a special rounded rod to get a very tightly packed joint *(Fig. 133)*.

➢ **The blocks must touch at the intrados. It is essential that the mortar joint has a triangular shape and that there is no thickness between the blocks at the intrados.**

4.4.3 Vaults & Domes Built with the Nubian Technique

The binder for vaults and domes is like a glue and should be more clayey than the mortar for walls in order to stick the blocks properly against each other. Nevertheless, this glue should not be too clayey, as it should not have excessive shrinkage, which can induce a lot of cracks in the structure later on. Note that soil and sand should be sieved with a 1 mm mesh.

- If the mortar for walls (1 cement: 4 soil: 8 sand) gives satisfactory results, the following mix can be successfully used for vaults and domes: 1 cement: 6 soil: 3 sand.
- If the mortar for walls is 1 cement: 3 soil: 9 sand, meaning that the soil is too clayey, the specification for vaults and domes could be 1 cement: 5 soil: 4 sand, or if needed, less soil and more sand.
- If the mortar for walls is 1 cement: 7 soil: 5 sand, meaning that the soil is too sandy, the specification for vaults and domes could be 1 cement: 7 soil: 2 sand, or if needed, more soil and less sand.
- If the soil is too sandy, no sand should be added and the mix could be 1 cement: 9 soil.
- If the soil is really too sandy and the mix 1: 9 does not give good results, the cement/soil ratio could be increased to 1 cement: 8 soil or 1cement: 7 soil, or even more.

The fluidity of the glue is essential for the adhesion. The fluidity and thickness of the glue varies according to the work:

- **Vaults, cloister and groin domes**
  - Fluidity:
    The glue needs to be very liquid. A sample of the glue taken with the trowel should leave a film of 3-4 mm thick on a trowel positioned vertically *(Fig. 119)*.
  - Thickness:
    The vertical joint, which binds the various courses of the vault, should be the minimum thickness. The best would be 1 mm thick and the maximum should be 2 mm thick.

Fig. 119 – 3-4 mm left on the trowel

- **Circular domes (Hemispherical, pointed and segmental)**
  - Fluidity:
    The glue needs to be semi liquid like a paste. A sample of the glue taken with the trowel should leave a film of 7-8 mm thick on a trowel positioned vertically *(Fig. 120).*
  - Thickness:
    The corners of the blocks touch each other at the intrados edge. As the courses are circular, the side of the joint facing the intrados has a triangular shape, which has changing proportions when the dome rises. It is crucial that the intrados corners of the block touch each other.
    (Fig. 148 and Fig. 149)

Fig. 120 – 7-8 mm left on the trowel

4.4.4 Vaults Built with the Free Spanning Technique

The Free Spanning technique, which employs horizontal courses, has been specially developed by AVEI for the construction of vaults without support. The binder is like a glue. Note that soil and sand should be sieved with a 1 mm mesh. The mortar specifications vary as the vault rises:

- The first courses, which are quite flat, need glue sandier than that for walls, in order to reduce the shrinkage when drying.
- When the courses rise, their angle becomes steeper. Therefore, the blocks tend to slip down and fall. The glue should increase the soil fraction, to increase the soil/sand ratio.

First courses of the vault

- If the mortar for walls (1 cement: 4 soil: 8 sand) gives satisfactory results, the first courses of the vaults, which are built with horizontal courses, can use this glue: 1 cement: 4 soil: 8 sand.
- If the mortar for walls is 1 cement: 5 soil: 7 sand, meaning that the soil is too clayey, the first courses of the vaults, which are built with horizontal courses, can use this glue: 1 cement: 3 soil: 9 sand, or less soil and more sand, if needed.
- If the mortar for walls is 1 cement: 7 soil: 5 sand, meaning that the soil is too sandy, the first courses of the vaults, which are built with horizontal courses, can use this glue: 1 cement: 9 soil: 3 sand, or more soil and less sand, if needed.

Higher courses of the vault

The fluidity of the glue is essential when laying the blocks. It should have the same fluidity as for the vaults built with the Nubian technique.

- When the courses become steeper and the blocks start to slip down, the glue should be more clayey. Add soil progressively to the glue and reduce the same proportion of the sand content.
- If the first courses use a mix of 1 cement: 4 soil: 8 sand, the glue can be modified as such: 1 cement: 5 soil: 7 sand, or more soil and less sand if needed.
- When the courses rise further and have a steeper angle, the soil/sand ratio should be increased progressively. At the top, the glue will have the same specification as that for vaults with the Nubian technique: 1 cement: 6 soil: 3 sand, or more soil and less sand, if needed.

Filling steps between courses

The extrados of an optimised vault, built with horizontal courses, has steps which should be filled with an earth concrete.

- If the mortar for walls (1 cement: 4 soil: 8 sand) gives satisfactory results, the mix for the earth concrete can successfully be: 1 cement: 2 soil: 3 sand: 4 gravel (1/2” size)

NOTE FOR ALL SPECIFICATIONS CONCERNING BINDERS

Types of soil are as different as human beings. Therefore, the various mixes which have been specified here are merely indicative and need to be adapted to suit each individual soil.

4.5 BUILDING ARCHES

Arches usually need a centring to be built. They may have any shape and span, but the blocks need a support on which to be laid. The main exception is for corbelled arches.

4.5.1 Centrings

Manufacture
Centrings can be made of wood, steel or masonry. Wood and steel centrings are useful when the same arch has to be built several times.

- Small wooden centrings can be made of waterproof plywood and can be very handy. Large wooden centrings are always heavy. They are mostly done by triangulation of wooden sections and their laying face is preferably done with a waterproof plywood.
- Wooden centrings are sensitive to moisture and may bend and twist. Their storage should be well taken care of in areas where there are termites.

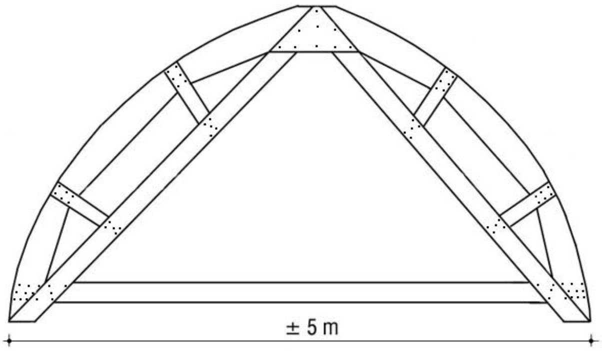


Fig. 121 – Wooden centring, ± 5 m span

- Steel centrings have the advantage of being light and not prone to damage by termites. Their storage requires only a place which is not exposed to rainwater. Steel centrings should be handled with care as shocks may easily deform them.
- The triangulation is preferably done with round rods of ∅ 6 or 8 mm, depending on the size of the arch. The laying face should never be finished with metal sheet but with two flat steel profiles (i.e. 25 x 6 mm) welded on either side upon round rods defining the curve.

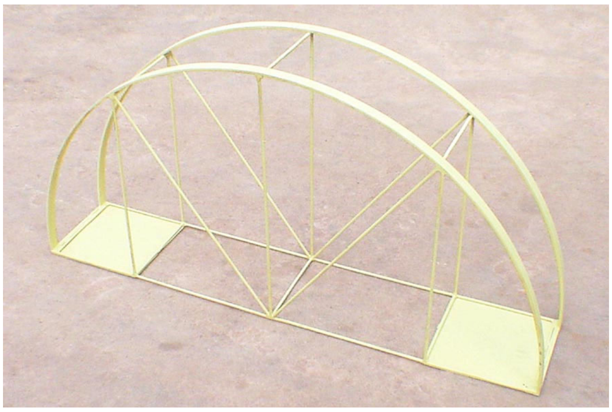


Fig. 122 – Steel centring, 90 cm span

- Masonry centrings are often used to save the cost of a prefabricated centring, as their cost is mostly the labour for its construction.
- They have to be dismantled after completion of the arch, and therefore are better suited when an arch has to be built only once.
- The masonry is laid with a very sandy mud mortar, which should not be stabilised, so that it can be scraped away easily.

Fig. 123 – Masonry centring, ± 80 cm span

The shape of the arch is first created approximately with blocks and then rounded with the sandy mud mortar. The laying face is preferably finished with a thin coat of cement stabilised earth mortar (i.e. 1 cement: 4 soil: 8 sand).

Adjusting the centring

Wooden and steel centrings can have supports made of wood poles or steel pipes only if the arch has to be built many times.

Most of the time, the supports are made with brickwork which is laid with a mortar made of earth and sand.

This mortar should be very sandy and without stabiliser, so that the supports can be dismantled easily afterwards. The top of the supports requires a flat surface to lay the wedges. These should be shaped according to the proportions shown in Fig. 131.

Follow this procedure once the supports have been laid at the proper height:

1. Lay the wedges in the 4 corners of the centring.
2. Gently place the centring above the wedges.
3. Load the centring with a few blocks *(Fig. 124)*.
4. Adjust the height, level and verticality of the centring by using a plumb line *(Fig. 125).*
5. This adjustment is done by sliding the wedges above each other *(Fig. 126)*.

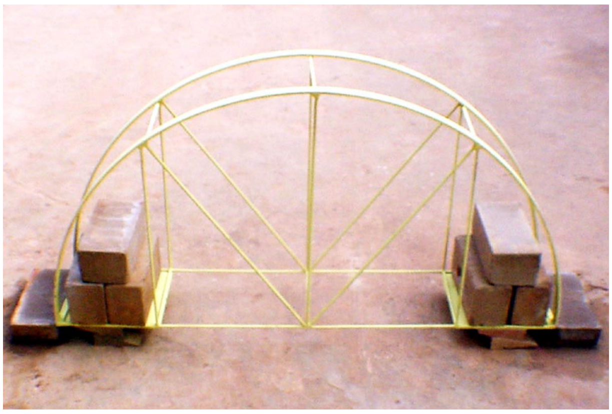


Fig. 124 – The centring is loaded with blocks

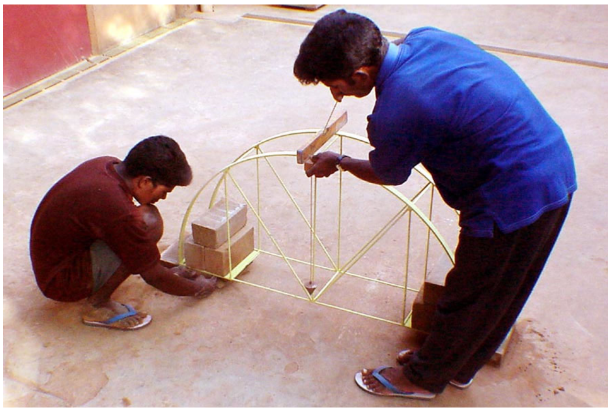


Fig. 125 – Check the level and verticality

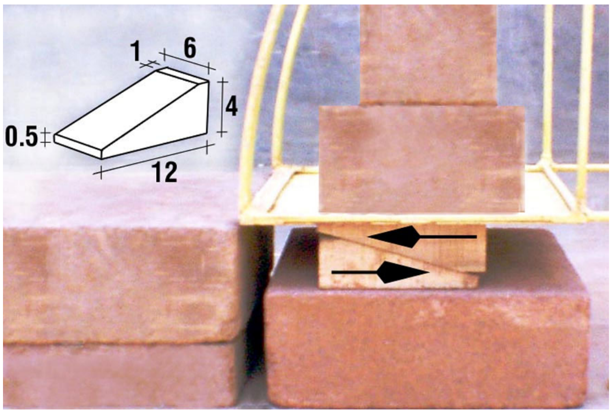


Fig. 126 – Adjusting the wedges (dimensions in cm)

4.5.2 Curved Arches with Centring

By “curved arches”, it is meant here arches which have a shape generated by one or several centre points (i.e. semicircular, pointed, bucket, etc.); this is not the case for corbelled arches.

The construction of curved arches requires a centring to support the voussoirs, unless it is built with the Free Spanning technique developed by the Auroville Earth Institute *(See Section 4.5.4: Arches with the Free Spanning Technique, p. 98)*.

Once the centring has been adjusted, the following steps should be executed. Note that for all types of arch, the general procedure for laying the blocks has common features, though depending on the arch shape, some details may vary slightly.

Common procedure for all arches

1. Lay the fluid mortar on the springer. It should have a triangular profile *(Fig. 127)*.

2. Soak a block in water and lay it on the springer. Check the right angle between the block and the centring.

3. Lay a block without mortar. Check the right angle between the block and the centring *(Fig. 128).*

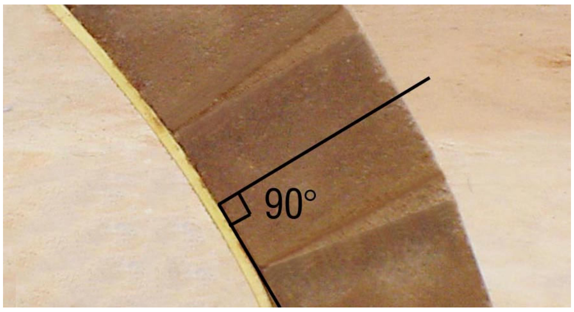


Fig. 127 – Triangular joint of the mortar

If the block is not laid perpendicularly, try to lay it by inversing its right and left sides: its thickness may not be even. When it is not possible to get a block perpendicular to the centring, it must be ground down to make it fit correctly.

4. When the block is perpendicular to the centring, lay fluid mortar on the previous block.

5. After soaking the block (previously checked), lay it on the centring and slide it down to compress the mortar.

6. Slide the block laterally to adjust it on the mortar be to get a triangular joint *(Fig. 129).* Check that the mid-point of the block is perpendicular to the centring *(Fig. 127)*.
It is essential that the blocks touch each other at the intrados. No mortar should be in between the blocks along the inside curve, and outside, the joint thickness will depend on the curvature of the arch.

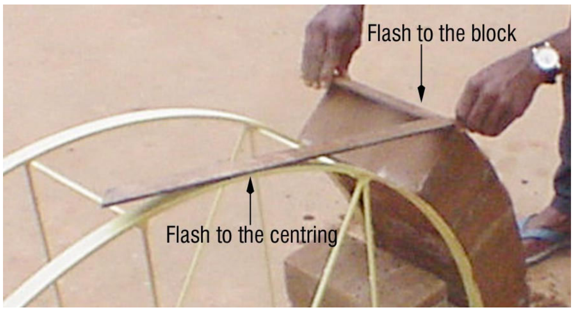


Fig. 128 – Check the right angle

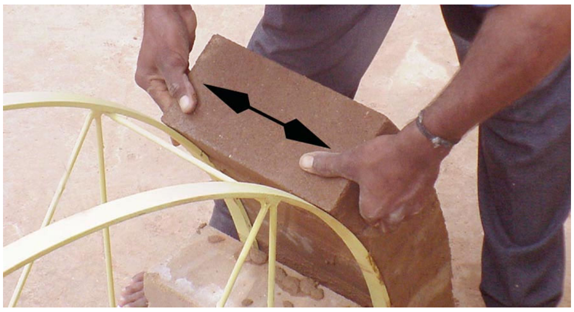


Fig. 129 – Slide the block laterally

7. Check that the blocks are aligned on one side of the centring (preferably outside).

8. Once a block is laid, always check the right angle between the block and the centring *(Fig. 128)*.
It is essential that the arch rises with the blocks perpendicular to the centring, so that the last blocks are parallel near the apex.

9. The arch must be built symmetrically, in order to balance the load on the centring and the masonry. Never have more than 2-3 blocks difference in height from one side to the other.

10. Laying the last blocks on top of the centring requires following a different procedure *(See Section 4.5.2: Curved Arches with Centring, Very flat segmental arches, p. 94)*, unless it is a pointed arch *(See Section 4.5.2: Curved Arches with Centring, Pointed arches, p. 94)*.

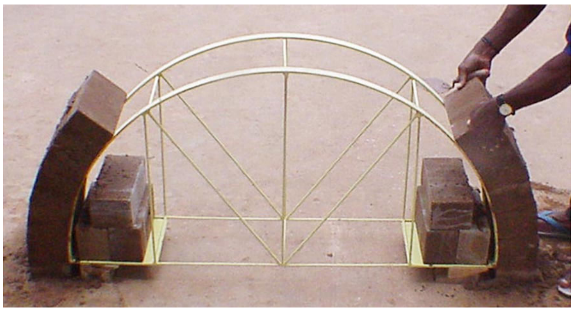


Fig. 130 – Build the arch symmetrically

11. Once the last block or the keystone has been laid and the brickwork cleaned, the centring can be removed: straight away for short spans (up to 3 m) and after half a day for longer spans.

12. Extreme care must be taken for the decentring. The centring should be dropped down slowly and vertically.

    Slide the wedges away a little bit simultaneously on both sides of one end, so that the centring drops down only by 3-4 mm. Do the same on the opposite end.

    Alternate slowly from one end to another, following the same action of sliding away the wedges until the wedges are totally removed.

    Be careful that the centring does not tilt and touch the arch anywhere from below.

13. The arch should be cured for one month after completion.

Fig. 131 – Removing wedges and decentring

Segmental arches

Depending on the flatness of the arch, the procedure will differ.

For arches which are not too flat, the blocks are laid on the side of the centring in a similar way to that described in the previous page *(See Section 4.5.2: Common procedure for all arches, p. 94).*

The last blocks laid on top of the centring are laid according to the details mentioned hereafter for very flat segmental arches.

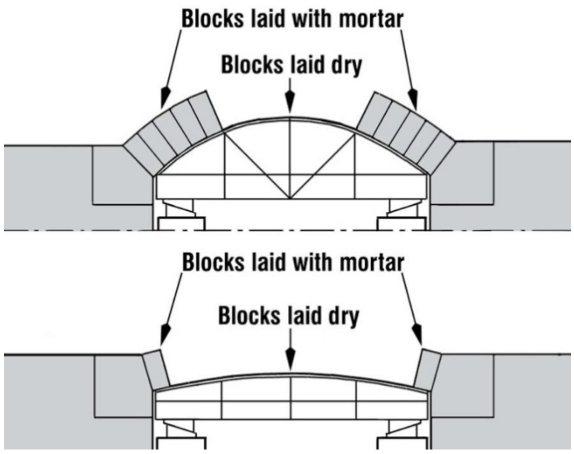


Fig. 132 – Roundness of segmental arches

Very flat segmental arches

These arches have a very narrow joint: 0 mm at the intrados (like the others), and sometimes only a few millimetres at the extrados.

Therefore, it is very difficult to achieve a precise joint, so that there is no mortar at the intrados. After laying the first block onto each springer, the blocks are laid dry without mortar, following this procedure:

1. Lay the fluid mortar on the springer. It should have a triangular profile *(Fig. 127)*.

2. Soak a block in water and lay it on the springer.

3. Check the right angle between the block and the centring *(Fig. 128).*

4. All the other blocks are laid dry on the centring – without mortar. Care should be taken so that the blocks touch at the intrados and are perpendicular to the centring.

5. Once the last block has been adjusted and inserted tightly, some water should be poured onto the blocks: the joints should be absolutely soaked.

6. Prepare very liquid grout by adding a lot of water to the fluid mortar. Pour it into the triangular joints: It should fill only the bottom part which is very narrow.

7. Prepare a mortar with a little water to fill the triangular joints: It should not be fully dry, though almost, and not plastic. This mortar should absorb the excess water of the liquid glue, which was required to fill the bottom part of the joint.

8. Once all the joints are filled, compress them with a bent rod, in order to exert pressure on the joint. The joint should be packed extremely tightly.

9. Once all joints are very tight, clean the brickwork and remove the centring *(See Section 4.5.2: Common procedure for all arches, p. 94)*.

10. The masonry should be cured for one month after completion.

Fig. 133 – Pressing the mortar joint

Semicircular arches
The procedure for laying the blocks is similar to the common procedure, up to the last blocks near the apex of the arch (3 to 7 blocks depending on the radius of the arch) *(See Section 4.5.2: Common procedure for all arches, p. 94)*. The last blocks are laid dry, following the procedure described for the very flat segmental arches.

Pointed arches
The procedure to lay the blocks is similar to the common procedure, up to the key stone *(See Section 4.5.2: Common procedure for all arches, p. 94)*. Pointed arches sometimes have a very narrow joint, so the mortar should be extremely fluid. The keystone is inserted tightly in between the last blocks.

Depending on the curvature of the arch, the keystone can be laid with very fluid mortar and adjusted precisely to the gap, or can be laid dry and a liquid mortar poured into the joint afterwards (as described for the flat segmental arches). In both cases, the keystone should be wedged tightly with stone chips in the joints.

4.5.3 Corbelled Arches without Centring

Corbelled arches were developed because they can be built without support, by regularly corbelling the horizontal courses of the wall masonry. The bond pattern is essential and the blocks should cantilever preferably by 1/4 of the block module, with the maximum projection of 1/3.

For building such an arch, it is essential to pay attention to the balance of the masonry as courses rise.

One should evaluate, before the masonry starts to tilt, where the centre of gravity is of the arch being built.

It should not go beyond the limit of stability, which is the inner side of the pier.

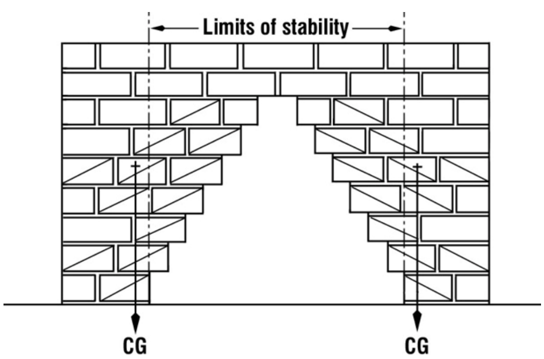


Fig. 134 – Centre of gravity of a corbelled arch

4.5.4 Arches with the Free Spanning Technique

A curved arch is normally never built free spanning, as it needs a centring to support the voussoirs. The following method was developed to build an arch without centring, in order to close a vault which was built with the Nubian technique, which starts from either ends of a room.

The Nubian technique needs a back wall to start sticking the vertical courses onto, and the vault is built arch after arch. At the other end it is nearly impossible to lay the last course between the vault and the opposite wall.

This technique was developed to start building the vault on both opposite walls at the same time. It presents the advantage of speed, as more masons can work on the same structure. As both halves of the vault become closer to each other, there will finally be a gap between both sides, which has to be closed. The method presented hereafter allows bridging this gap between both halves of the vault – without support.

1. It is essential to end with a space wider than the block length, by 3-5 mm. This should be planned in advance and controlled carefully during construction *(Fig. 136)*. To calculate the number of courses, their total length and how it can be adjusted, you can assume 51.5 mm for the width of a course (50 mm for the block thickness + 1.5 mm for the thickness of the glue on average).

2. Begin sticking the blocks on both opposite walls at the same time and build the vault in the normal way with the Nubian technique *(Fig. 135)*.

Fig. 135 – Start the vault on both sides

3. Check regularly that the vault is progressing as planned: control the length of both halves, to leave the required gap for the free spanning arch.

4. From time to time, adjust the length of both halves according to the calculation:
   - Grind a course if the vault portion is slightly too long.
   - At the end of the working day, apply a thin coat of very sandy stabilised earth plaster i.e. 1: 4: 8 (same mix as for arches) if the vault portion is too short: a few millimetres only to compensate for the length which is missing.

5. This adjustment is often needed at the end to achieve the proper gap. The space left between the final opposing courses should be 2-3 mm more than the length of the block to be inserted. The last course can be slightly ground to have the proper size gap. Note that it is always better to grind the last course of the vault rather than to plaster it if the gap is too large.

6. To start building the free spanning arch, soak a block in water and apply some glue (1 cement: 9 soil: 3 sand) onto the block to stick it on the springer *(for glue specifications, See Section 4.4: Binder Quality, p. 90)*. The glue should be 2-3 mm thick only.

7. Grind a block to adjust its length if required *(Fig. 137).*

   Soak the block and apply some glue onto it. Level the glue to have only 2-3 mm *(Fig. 138)*.

8. Wet the previous course if it is already dry.

9. Stick the new block onto the previous course and slide it gently up and down to compress the glue *(Fig. 139 and Fig. 140)*.

Fig. 136 – Check the linearity of the last course

10. Insert a stone chip on both side of the block laid and the last course of the vault. It should be very tight *(Fig. 141)*.

11. Adjust the keystone by grinding it in such a way that it is tightly fitted at the intrados and with 2-3 mm play on top, to fit in the gap *(Fig. 142).*

12. Pour some water on the keystone and apply some glue on the four laying faces *(Fig. 143 and Fig. 144).*

Fig. 137 – Grind a block to adjust its length

13. Insert the keystone. It should be a tight fit at the intrados and wedge itself *(Fig. 145)*.

14. Hit gently the keystone gently to get a tight fit *(Fig. 146).*

15. Wedge the keystone with stone chips:
    - On both sides in the joint of the previous blocks of the arch.
    - On both sides in the joint between the arch and the last course of the vault *(Fig. 147).*

16. The masonry should be cured for one month after completion.

Fig. 138 – Apply 2-3 mm of glue on the block

Fig. 139 – Insert the block. Note the mortar on the sides

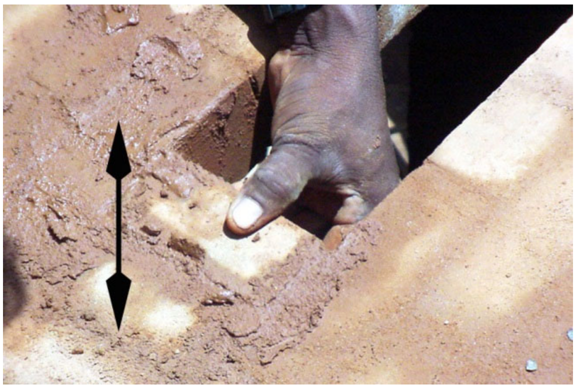


Fig. 140 – Adjust the block by sliding it vertically

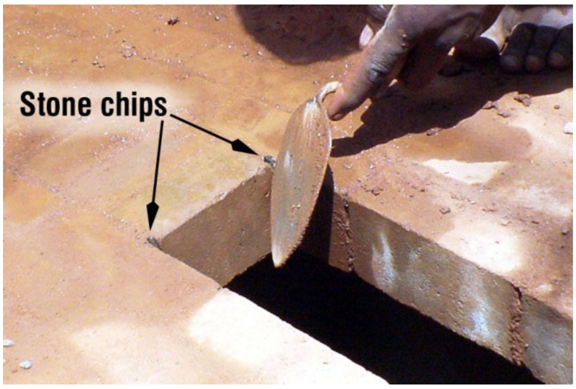


Fig. 141 – Wedge the block with stone chips

Fig. 142 – Grind the keystone to adjust its thickness

Fig. 143 – Pour water on the keystone

Fig. 144 – Apply 2-3 mm of glue on the 4 laying faces

Fig. 145 – Insert the keystone

Fig. 146 – Gently hit the keystone to wedge it into place

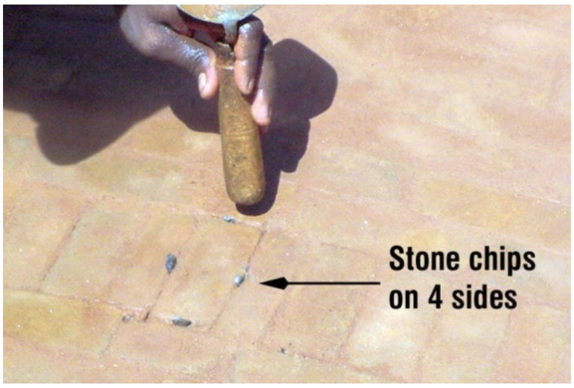


Fig. 147 – Wedge the keystone with stone chips

4.6 BUILDING VAULTS

4.6.1 Building a Vault with the Nubian Technique

The back wall should be built first. It can have exactly the shape of the extrados of the vault or it can be quadrangular and the extrados of the vault will be drawn onto it *(Fig. 148)*.

A template is needed to ensure the proper shape of the vault. It can advantageously be the future window frame *(Fig. 149)* on which are temporarily fixed some spacers to control the extrados shape of the vault. The template can also be made of welded Tor steel, which can be re-used afterwards for reinforced cement concrete.

Fig. 148 – Back wall

It is necessary to create a network of string lines between the back wall and the template. Note that it is better to lay the net of string lines outside of the masonry. The reason is that any mistake in accuracy, e.g. a block laid lower or slipping down, will not change the linearity of the string line.

In certain cases, it is sometimes necessary to lay the string lines below the masonry. It is then indispensable to work with a very high accuracy and to always leave 1 mm gap between the blocks and the string line.

Fig. 149 – Window as a template

Note that it is essential that the blocks are absolutely dry before beginning the vault. The following procedure must be followed to build the vault properly:

1. Build the back wall with the extrados shape or draw the extrados of the vault onto it.

2. Set up the template on the opposite side and brace it properly so that it can withstand the tensile force of the string lines.

3. Stretch the string lines (nylon, 1mm diameter) very firmly from the wall to the template. The spacing between them needs to be preferably the same as the block length or 14 to 20 cm. The string lines should be nailed in the quadrangular wall or hooked on the wall shaped with the vault extrados.

4. Pour some water onto the back wall and briefly soak a totally dry block in water. This will begin capillary suction, which will continue with the clay of the glue once it has been applied.

5. Lay some glue (1 cement: 9 soil: 3 sand) onto the block *(for glue specifications, See Section 4.4: Binder Quality, p. 90)*. It should be 2-3 mm thick only.

6. Stick the block immediately against the wall. The block should be stuck a few millimetres higher than its position and slid down while pressing the block.

7. It is essential that the bottom corners of the blocks touch each other. No mortar should be left in between them at the intrados. The thickness of the joint at the extrados will depend on the curvature of the vault.

8. Once a block is stuck, wedge the top part of the joint with a stone chip. It should be tight, to transmit the compression forces, especially during construction. Be careful not to move the block while inserting the stone chip.

9. Check that no mortar or block touches the string lines: keep always 1 mm gap between them.

10. Note that the vault has to be built symmetrically. This means that if it is a small span, the mason should lay the blocks alternately from left to right. If the span is large, two masons or more can work simultaneously on both sides.

11. The length of the last blocks of a course will have to be adjusted to fit in the space remaining on top.

12. When starting the second course, don't forget to cross the bonds: the first block of the second course is half the length of the one from the first course. Similarly, every new course will start with a difference of half a block from the previous course.

13. Check regularly that nothing touches the string lines, which should be absolutely straight.

14. When the vault is nearing completion, the template will disturb the mason. Therefore, it should be removed and the work continues with a straight edge to ensure the proper shape of the vault.

15. The masonry should be cured for one month after completion.

4.6.2 Building a Vault with the Free Spanning Technique

We have seen that this technique allows one to lay courses horizontally and that it also employs vertical courses, like in the Nubian technique. What is presented here is the particular details for laying the courses horizontally.

The binder, as described in Section 4.4: Binder Quality, p. 90, varies as the vault rises. It starts with the same specification as for arches and progressively becomes more clayey.

It is essential to check the balance of the portion of the vault which progressively cantilevers. Therefore, to ensure the height of the various courses, their cord and span must be checked to ensure that they are according to the calculations.

Follow the procedure described here to build a vault with horizontal courses:

1. Stretch the net of string lines from the wall to the template.
   Note that for vaults built with horizontal courses, the string lines will be placed on the intrados side. This is compulsory, as the thickness of the vault varies when it rises. Therefore, it is essential to always keep a 1 mm gap between the string line and the blocks.

2. Pour water onto the springer and then soak a block in water.

3. Use stabilised earth glue 1 cement: 4 soil: 8 sand and apply some on the block *(for glue specifications, See Section 4.4: Binder Quality, p. 90)*. The thickness of the glue should be just a few millimetres at the intrados side and, at the extrados side, more than the thickness of the finished joint.

4. Lay the block and slide it gently back and forth, to adjust the mortar thickness and the angle of the block. No mortar should be left at the intrados: the block should be touching the springer, and at the extrados, the joint thickness will vary with the curve.
   Ensure that 1 mm separates the blocks and the string line. The blocks must never touch the line.

5. Apply some glue (3-4 mm thick) on the side of the block which has just been laid. Lay the next block against it. The vertical joint will be compressed up to 1-2 mm by hammering it gently on the side *(Fig. 150)*.

It is essential to compress the vertical joint very well and to keep it to the minimum, in order to reduce the shrinkage of the glue and cracks which can later develop in the vault.

6. Continue laying the blocks in the same way for the entire course, and meanwhile remembering that the vault has to be built symmetrically.

Fig. 150 – Compress the joint

7. Do not forget to establish a bond pattern from course to course, while laying the next courses.

8. When the bond of the course is 1.5 or 2 blocks wide, the vertical joint which runs all along the course should be of the desired thickness (not more than 1 cm) and compressed with a rod.

9. The glue quality needs to change as the courses of the vault rise:
   When the angle of the course becomes steeper, the blocks will tend to slip down. Therefore, the glue should become more clayey. Add some soil progressively to the glue and reduce the sand content by the same proportion.
   - If the first courses use a mix of 1 cement: 4 soil: 8 sand, the glue can be modified as such: 1 cement: 5 soil: 7 sand, or more soil and less sand if needed.

   When the courses rise further and have a steeper angle, the soil/sand ratio must be increased even more. The glue will have at the end the same specification as that for vaults with the Nubian technique: 1 cement: 9 soil: 3 sand, or more soil and less sand if needed.

10. When the last course of a row is over, check if its height is according to the calculations:
    - Check the cord of the portion, from the springer intrados to the inner edge of the course which has been completed.
    - Check the span of the course, from the inner edges of the top block of the course.

11. If it is not satisfactory, some adjustments should be made to keep the proper width at the correct height:
    - If the thickness is wider than it should be, the course is higher than the calculations:
      Grind the extrados edge of the blocks and ensure that the next rows of courses do not add to the difference. If they do, it might be necessary to remove one course in a row after some time, in order come back to the proper width at the correct height.
    - If the thickness is not as wide as it should be, the course is lower than the calculations:
      Mark a reference on the extrados edge of the top block of the course, showing what the correct thickness should be, so it can be filled with the earth concrete.

    It is essential to ensure that the width of the course, at a given height, is identical to the calculations.

12. As soon as a set of courses with the same width is completed, the step between it and the previous one should be filled with an earth concrete: 1 cement: 2 soil: 3 sand: 4 gravel (1/2” size)

13. Once the last course of the last set of the horizontal courses is completed, the vault continues with vertical courses.

4.7 BUILDING DOMES

4.7.1 Circular Domes

Circular domes are defined by the rotation of a compass. The length of the compass is taken at the outer diameter of the dome, so that the direction of the block can be adjusted by the angle of the compass. The control of the shape is ensured from the inner diameter and thus a cursor or any kind of mark made on the compass is required.

The procedure described as follows is for any kind of circular dome built on a circular plan.

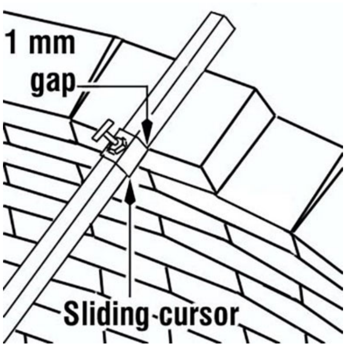


Fig. 151 – Compass

Fig. 152 – Building hemispherical dome on pendentives

Fig. 153 – Checking blocks with a compass

1. Fix the compass at the centre of the room at the required height. The compass should be loaded with blocks, to ensure that it does not to move during construction.

2. Soak a totally dry block briefly in water.

3. Lay some glue on the block *(for glue specifications, See Section 4.4: Binder Quality, p. 90)*. The glue will be laid with a triangular shape from inside to outside, to follow the joint profile *(Fig. 154)*. Note that the thickness of the glue will vary as the dome rises: the horizontal joint will become more triangular as the radius diminishes *(Fig. 155)*.

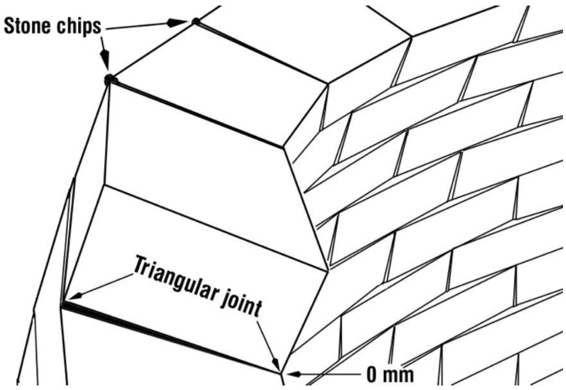


Fig. 154 – Triangular shape of the mortar (section)

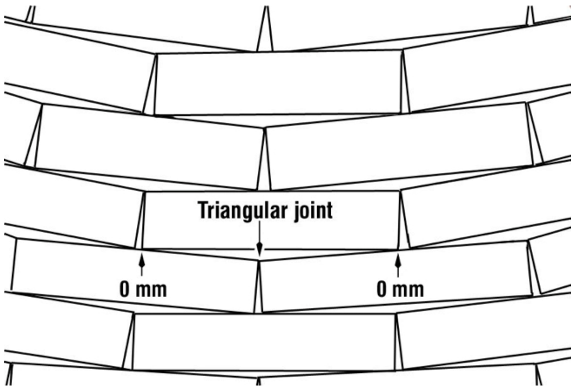


Fig. 155 – Triangular shape of the joint (inside)

4. Adjust the block by pressing it on the ring beam or the wall:
   - Slide it slightly, until there is no mortar at the intrados corners. The extrados thickness and block inclination will be adjusted by following the direction of the compass.
   - The radius will be checked by keeping 1 mm gap between the cursor of the compass and the block.
   - The centre of the block should be perpendicular to the compass.

5. Once the first block is adjusted, lay some mortar on one of its sides, and repeat the procedure for laying the block:
   - Soaking a block in water.
   - Laying some glue. Note that the first courses have a flat joint (viewed from inside), and as the dome rises, it becomes progressively more triangular *(Fig. 155)*.
   - Adjusting the block as described above.

6. Remember that the blocks must touch at the intrados: the bottom corners touch the previous course, and the top corners touch the blocks adjacent to it *(Fig. 154 and Fig. 155)*.

7. Once a block has been laid, the joint at the extrados side should be wedged with a stone chip, to transmit the compression forces *(Fig. 154)*.

8. Do not forget to establish a bond between the courses. It should be around half the length of the block.

9. As the dome rises, the diameter of the rings diminishes and the length of the blocks needs to be regularly adjusted to provide a bond with a half cover from the previous course.

4.7.2 Square Domes

Square domes are generated by the intersection of two vaults, which create the groin or cloister domes. The procedure described as follows is for cloister domes which are built with squinches.

1. A template is required and it is generally made of a pipe which is bent according to the need. Fix it properly on the ring beam. Load it with blocks in the corners, to prevent it from moving.

2. Pull the string lines at regular intervals, from diagonal to diagonal of the template. The spacing between the string lines can be the block length or 10 to 14 cm.

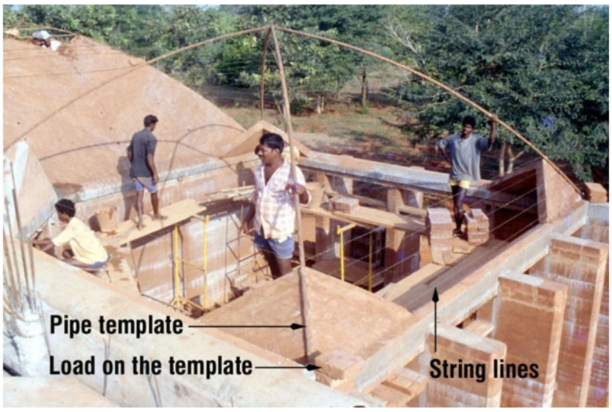


Fig. 156 – Pipe template and string lines

3. Begin the squinches by filling the corner with an earth concrete, into which are inserted small pieces of broken blocks. This filling will be perpendicular to the template and at 45° from the springer beams.

4. Soak a totally dry block briefly in water.

5. Lay a thin and flat coat of glue onto the block. It will be around 2 to 3 mm thick *(for glue specifications, See Section 4.4: Binder Quality, p. 90)*. Once pressed by the block, the glue will be preferably 1 mm thick mm and at the maximum 2 mm thick.

6. Stick the block immediately. The blocks of each new arch, which are laid on both sides of the springer beam, should be wedged with a small piece of a broken block.

7. Both sides of the squinche should be built alternately, in order to balance the masonry.

8. Do not forget that the blocks must touch each other at the intrados. The thickness of the joint at the extrados will vary with the curvature of the dome.

9. Once a block is stuck, wedge the extrados side of the joint with a small stone chip. This will transfer the compression forces and will avoid the slippage of blocks and the deformation of the arch.

10. The two last blocks which close each arch will become the keystone. They should be laid in such a way that a bond will be established between odd and even course:
    The vertical joint will be alternately placed on the right and left side of the vertical axis, in order to cross the bonds.

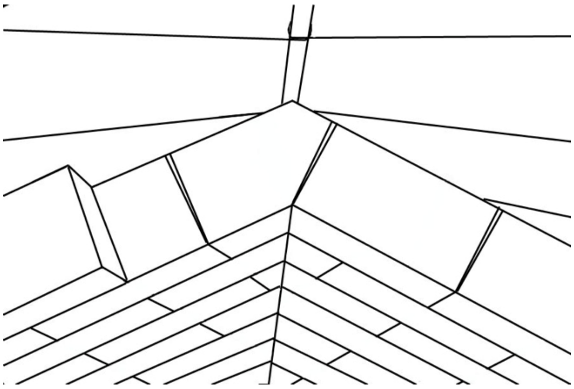


Fig. 157 – Alternately cross the blocks for the keystone (left)

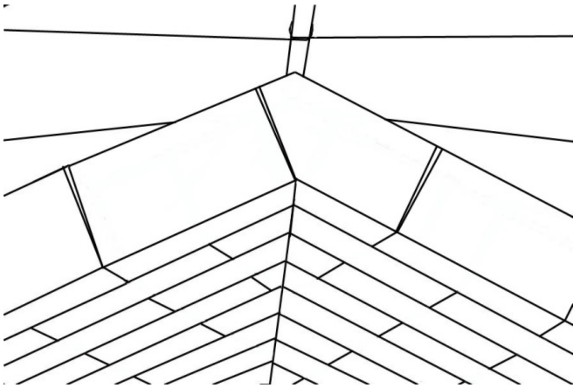


Fig. 158 – Alternately cross the blocks for the keystone (right)

11. As soon as an arch is closed, another one can be begun straight away, but do not forget that the squinches should be built symmetrically.

12. When the four squinches meet at the centre of the springer beams, the arches of each squinche will alternately be crossed in a herringbone pattern.

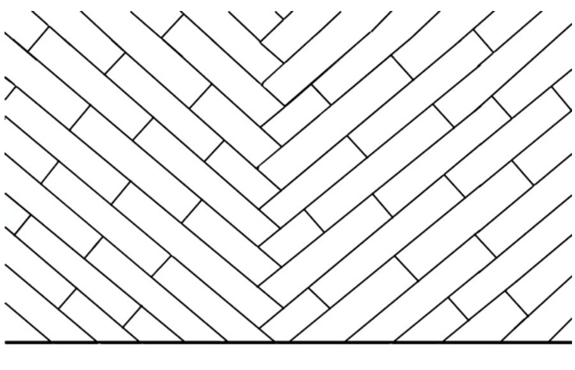


Fig. 159 – Herringbone pattern of the groin, where the squinches meet at the mid-span

13. As the blocks are laid below the string lines, check throughout construction that no mortar or block touches the string lines: keep always 1 mm gap between them.

5. ANNEXES

- $x = R \cdot \cos\alpha - E$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\frac{\alpha}{2}\right)$
- $\alpha = \frac{180 \cdot A_{Intrados}}{\pi R}$
- $A_{Intrados} = \frac{\pi R \alpha}{180}$
- $R = \sqrt{r^2 + E^2}$
- $r = \sqrt{R^2 - E^2}$
- $s = R - E$
- $E = \sqrt{R^2 - r^2}$

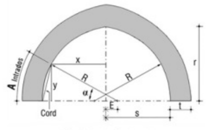


Fig. 170 – Pointed arch

- $x = R \cdot \cos\alpha$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\frac{\alpha}{2}\right)$
- $\alpha = \frac{180 \cdot A_{Intrados}}{\pi R}$
- $A_{Intrados} = \frac{\pi R \alpha}{180}$
- $A_{Extrados} = \pi(R + t)$
- $Area_{Arch} = \frac{\pi t(2R + t)}{2}$

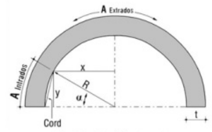


Fig. 169 – Semicircular arch

- $R = \frac{r^2 + s^2}{2r}$
- $r = R - \sqrt{R^2 - s^2}$
- $s = R \cdot \sin\left(\frac{\alpha}{2}\right)$
- $s = \sqrt{2rR - r^2}$
- $A_{Intrados} = \frac{\pi R \alpha}{180}$
- $A_{Extrados} = \frac{\pi(R + t)\alpha}{180}$
- $\sin\left(\frac{\alpha}{2}\right) = \frac{s}{R}$
- $\alpha = \frac{180 \cdot A_{Intrados}}{\pi R}$
- $\beta = 90 - \left(\frac{\alpha}{2}\right)$
- $\tan\beta = \frac{(R - r)}{s}$
- $Area_{Arch} = \frac{\pi \alpha t(2R + t)}{360}$

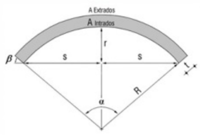


Fig. 168 – Segmental arch

- $x = R \cdot \cos\alpha - \frac{R}{2}$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\frac{\alpha}{2}\right)$
- $\alpha = \frac{180 \cdot A_{Intrados}}{\pi R}$
- $R = 2s$
- $r = R \cdot \sin 60°$
- $A_{Intrados} = \frac{\pi R \alpha}{180}$

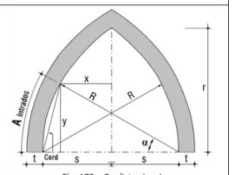
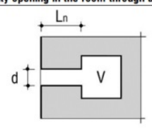
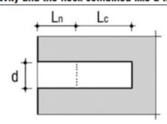


Fig. 172 – Equilateral arch

Cavity opening in the room through a neck		Cavity and the neck combined like a tube	
 Fig. 162 – Cavity Resonator (with a cavity and neck)		 Fig. 163 – Tubular Resonator	
Formula 1 $V = \frac{(343d)^2 \times 10^4}{16\pi f^2 (L_n + 0.8d)}$	**Formula 2** $f = \frac{343 \times 100}{2\pi}\sqrt{\frac{d^2}{4V(L_n + 0.8d)}}$	**Formula 1.1** $Lc = \frac{343^2 \times 10^4}{4\pi^2 f^2 (L_n + 0.8d)}$	**Formula 2.2** $f = \frac{343 \times 100}{2\pi}\sqrt{\frac{1}{L_c(L_n + 0.8d)}}$

Process for calculating the cavity volume

1. Select the frequency to be absorbed (f)
2. Define the neck diameter (d) and length (L_n)
3. Apply Formula 1.
4. Check using Formula 2 that the frequency absorbed is the targeted frequency.

Process for calculating the tube length

1. Select the frequency to be absorbed (f)
2. Define the neck diameter (d) and length (L_n) Ln is required to begin the calculation. It should be relatively short (~10, 15 cm).
3. Apply Formula 1.1 to calculate the remaining tube length (L_c)
4. Add $L_n + L_c$ to get the length of the entire tube.
5. Check using Formula 2.2 that the frequency absorbed is the targeted frequency.

Bucket or Basket handle arch
(Arc en anse de panier)
An arch composed of 3 or more centres: 2 centres are located on the springer line, symmetrical to the axis. 1 centre is located on the axis, below the springer line. Bucket arches can have 5, 7 or more centres (always an odd number). An ellipse can be considered as a bucket arch, which has an infinite number of centres.

Buttress *(Contrefort)*
A rectangular column of masonry typically either integrated into a wall in its entirety, or integrated into the wall only at its base. It can also be free-standing. It receives and balances the lateral thrust of a vault.

Catenary Arch *(Arc en chainette)*
The curve assumed by a freely suspended chain or flexible cable under the action of gravity. The centre line of the links is the line of tensile stress.
In an arch which has the shape on an inverted catenary curve, the voussoirs correspond to the links of the chain. Just as the links of the chain are only in tension, the voussoirs of the inverted catenary arch are only in compression. The curve taken by the chain corresponds to the line of thrust of the arch. Catenary arches are always the most stable and their thickness can be minimized.

Centring *(Cintre)*
Wooden or steel support for building an arch or vault, which is removed after completion.

Clerestory *(Lanterneau)*
The upper part of the nave, choir and transepts, containing a series of windows above the roofs of the aisles.

Cloister Vault or Cloister Dome
(Voûte ou dôme en arc de cloître)
A four-sided dome, which consists of portions of the surfaces of two barrel vaults, often pointed, that intersect perpendicularly. The diagonal arches of the cloister dome are identical to the groins of the groin vault, which have the same generating geometry. The cloister dome does not show the cross section of the barrel vault. The thrusts are continuous along the entire perimeter of the supporting walls, rather than confined to the corner piers, as in the groin vault.

5.1 ACOUSTICS & ACOUSTIC CORRECTORS

5.1.1 Acoustics of Vaulted Structures

Vaulted structures have two acoustic phenomena: echo and reverberation. *Echo* is the phenomenon of reflected sound, which can be perceived as physically disturbing. *Reverberation* is the interpretation of the persistence of a sound in a particular space, or a large number of reflections which build up and decay.

Echo occurs only in domes which are a geometrical segment of a sphere. The dome may be built on a circular plan or on a quadrangular one (i.e. dome on pendentives), but it will always have an echo if it has the shape of the segment of a sphere or a shape close to this. Hemispherical domes have the most pronounced echo. Pointed domes rarely have any echo, but they can have high reverberation and/or amplify sound. Domes generated by the intersection of vaults (i.e. cloister or groin dome) do not have any echo at all.

Vaults and domes always manifest high reverberation, which represents the time needed for the sound to fade away. This reverberation occurs as a function of three primary factors:

- The size of the volume created by the vaulted structure, which is generally larger than adjacent volumes.
- The shape of the structure, and the propensity for this shape to reflect sound.
- Materials used for the walls and vaulted structures.

5.1.2 Acoustic Correction with Single Resonator Absorbers (Helmholtz Resonator)

Echo and reverberation can be limited by acoustic correctors, called *single resonator absorbers* or Helmholtz resonators. These types of absorbers, typically ceramic pots on the backs of vaults, have been traditionally used in Islamic mosques and European monasteries to correct the acoustics of vaults and domes. Formulas elaborated by the German engineer Hermann von Helmholtz allow for the calculation of single resonator absorbers.

The principle of this kind of resonator absorber is that a small, self-enclosed cavity with a port or neck opening into a space, will resonate at a certain frequency. When a sound wave traveling through space strikes the resonator, the pressure within the cavity changes and the air within the resonator neck vibrates. This causes sound to be absorbed by viscous loss (or damping).

Such resonator absorbers can take two different forms: The cavity can have a larger diameter than the neck port *(Fig. 160)*, or the cavity and neck can have the same diameter, making it tubular *(Fig. 161)*.

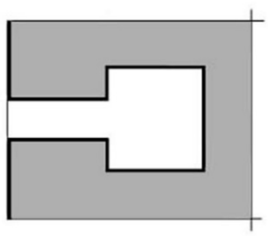


Fig. 160 – Cavity Resonator (with a cavity and neck)

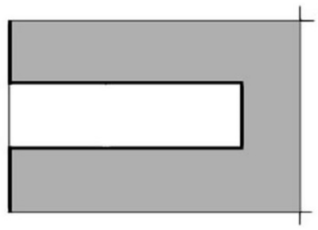


Fig. 161 – Tubular Resonator

The human voice ranges from 100 Hz to 2 KHz, with an average frequency at 400 Hz. A single resonator is effective only for a limited range of frequencies, which are close to the resonant frequency at which it shows a sharp peak. Correctors of various sizes can be installed to absorb the desired frequencies. The frequency absorbed is related to the volume of the cavity and the diameter and length of the neck of the resonator. The larger the cavity is, the lower the absorbed frequency will be.

When several single resonator absorbers are combined, in order to absorb a wide spectrum of frequencies, these must not be multiples of each other. For example, if one wants to absorb frequencies of 200 and 400 Hz, the result will not be effective; it would be better to target frequencies of e.g. 220 and 380 Hz.

5.1.3 Simplified Formulas to Calculate a Single Resonator Absorber

The formulas are such that the neck length and diameter have to be given as separate values. These formulas are only for a neck which is circular. Note that for the tubular resonator absorber, the neck length is a theoretical value, as the resonator is cast as a single tube.

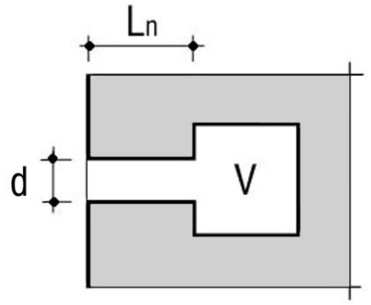
<table>
<tr><th colspan="2">Cavity opening in the room through a neck</th><th colspan="2">Cavity and the neck combined like a tube</th></tr>
<tr><td colspan="2">

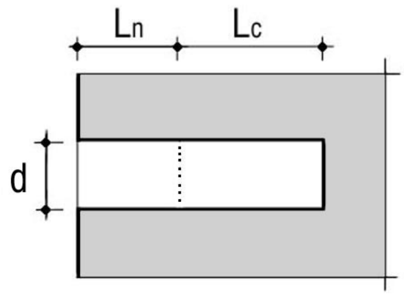
Fig. 162 – Cavity Resonator (with a cavity and neck)</td><td colspan="2">

Fig. 163 – Tubular Resonator</td></tr>
<tr><td>Formula 1
$V = \frac{(343d)^2 \times 10^4}{16\pi f^2 (L_n + 0.8d)}$</td><td>Formula 2
$f = \frac{343 \times 100}{2\pi} \sqrt{\frac{d^2}{4V(L_n + 0.8d)}}$</td><td>Formula 1.1
$Lc = \frac{343^2 \times 10^4}{4\pi^2 f^2 (L_n + 0.8d)}$</td><td>Formula 2.2
$f = \frac{343 \times 100}{2\pi} \sqrt{\frac{1}{L_c(L_n + 0.8d)}}$</td></tr>
<tr><td colspan="2">Process for calculating the cavity volume
1. Select the frequency to be absorbed (f)
2. Define the neck diameter (d) and length (Ln)
3. Apply Formula 1.
4. Check using Formula 2 that the frequency absorbed is the targeted frequency.</td><td colspan="2">Process for calculating the tube length
1. Select the frequency to be absorbed (f)
2. Define the neck diameter (d) and length (Ln)
Ln is required to begin the calculation. It should be relatively short (~10, 15 cm).
3. Apply Formula 1.1 to calculate the remaining tube length (Lc)
4. Add Ln+Lc to get the length of the entire tube.
5. Check using Formula 2.2 that the frequency absorbed is the targeted frequency.</td></tr>
</table>

Where:

f	V	Ln	Lc	d
Frequency (Hz)	Cavity volume (cm^3)	Neck length (cm)	Cavity length (cm)	Hole diameter (cm)

343 is ± the sound velocity at 20 °C temperature and 1013 H Pa atmospheric pressure.
(Maekawa, 1994)

5.1.4 Example of Single Resonator Absorbers & Frequencies Absorbed

Pipe ∅ (cm)	Neck length (cm)	Desired frequency (Hz)	Cavity length (cm)	Frequency absorbed (Hz)	Pipe length (cm)
9	55	120	33	120.5	88
	55	220	10	218.9	65
	55	320	5	309.6	60
	55	420	2.5	437.8	57.5
5	10	520	8	518	18.
	10	620	5.5	622	15.5
	10	720	4	729.5	14.
	10	820	3	842.3	13
4	10	920	2.5	950.3	12.5
	10	1020	2	1062.5	12
	10	1120	1.7	1226	11.5
	10	1220	1.5	1502	11

This system was implemented in the dome of the Dhyanalinga Temple, which consequently has no echo. Nevertheless, the dome still has some reverberation, which is predictable on account of its size (22.16 m diameter with a rise of 7.90 m and a total height of 9.70 m).

The absorbed frequencies are slightly different than the targeted values, because the holes for these resonator absorbers were cast with PVC pipes. Therefore, the hole diameters had to be adapted to the PVC pipe sizes which were available on the market. These pipes are removed after casting.

Fig. 164 – Inserting a pipe of 65 cm long

Fig. 165 – Inserting a pipe of 15.5 cm long

Fig. 166 – Protecting a resonator of 88 cm long

Fig. 167 – Closing a resonator of 88 cm long

5.2 GEOMETRIC FORMULAS

SEGMENTAL ARCH

- $R = \dfrac{r^2 + s^2}{2r}$
- $r = R - \sqrt{R^2 - s^2}$
- $s = R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $s = \sqrt{2rR - r^2}$
- $A_{Intrados} = \dfrac{\pi R \alpha}{180}$
- $A_{Extrados} = \dfrac{\pi (R + t)\alpha}{180}$
- $\sin\left(\dfrac{\alpha}{2}\right) = \dfrac{s}{R}$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $\beta = 90 - \left(\dfrac{\alpha}{2}\right)$
- $\tan \beta = \dfrac{(R - r)}{s}$
- $Area_{Arch} = \dfrac{\pi \alpha t (2R + t)}{360}$

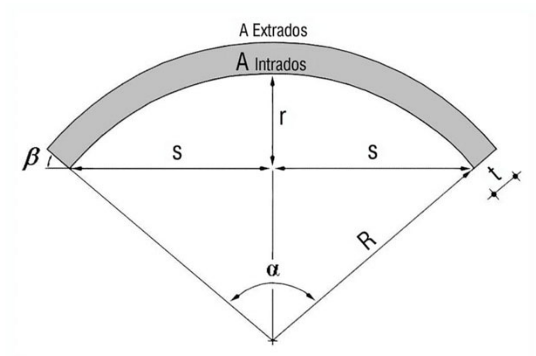


Fig. 168 – Segmental arch

Where: R=Radius, r=Rise, s=1/2 span, t=Thickness
$A_{Intrados}$=Length of the arch inside
$A_{Extrados}$=Length of the arch outside

SEMICIRCULAR ARCH

- $x = R \cdot \cos\alpha$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $A_{Intrados} = \dfrac{\pi R \alpha}{180}$
- $A_{Extrados} = \pi(R + t)$
- $Area_{Arch} = \dfrac{\pi t (2R + t)}{2}$

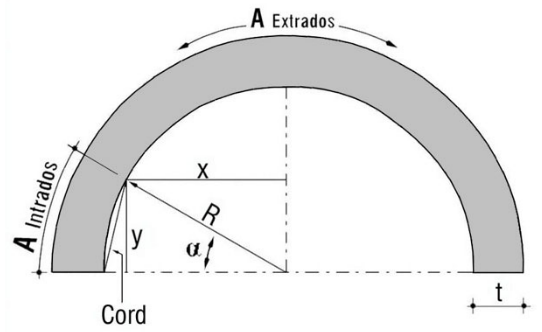


Fig. 169 – Semicircular arch

Where: R=Radius, C=Cord, t=Thickness
$A_{Intrados}$=Number of blocks x block height
$A_{Extrados}$=Length of the arch outside

POINTED ARCH

- $x = R \cdot \cos\alpha - E$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $A_{Intrados} = \dfrac{\pi R \alpha}{180}$
- $R = \sqrt{r^2 + E^2}$
- $r = \sqrt{R^2 - E^2}$
- $s = R - E$
- $E = \sqrt{R^2 - r^2}$

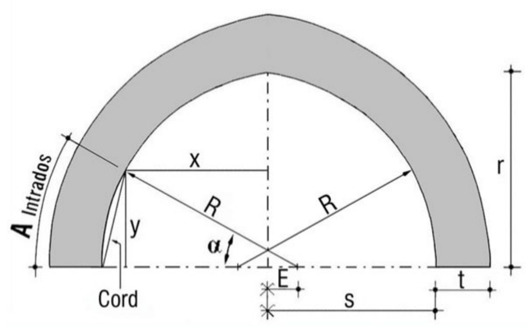


Fig. 170 – Pointed arch

Where: R=Radius, r=Rise, s=1/2 span, t=Thickness
C=Cord, E=Eccentricity
$A_{Intrados}$=Number of blocks x block height

SEGMENTAL POINTED ARCH

- $S_1 = R - E$
- $s = R \cdot \cos\alpha - E$
- $b = \sqrt{R^2 - E^2}$
- $r = E \cdot \tan\beta - a$
- $r = R \cdot \sin\beta - a$
- $R = \sqrt{(r+a)^2 + E^2}$
- $R = \sqrt{(s+E)^2 + a^2}$
- $\sin\alpha = \dfrac{a}{R}$
- $\cos\beta = \dfrac{E}{R}$
- $x = R \cdot \cos(\alpha + \gamma) - E$
- $y = R \cdot \sin(\alpha + \gamma) - a$
- $C = 2R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $A_{Intrados} = \dfrac{\pi R \gamma}{180}$

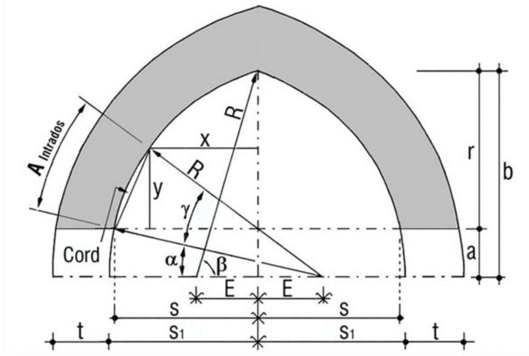


Fig. 171 – Segmental pointed arch

Where: R=Radius, r=Rise of the segmental pointed arch, s=1/2 span of the segmental pointed arch
S_1=1/2 span of the actual pointed arch, C=Cord, b=Rise of the actual pointed arch
t=Thickness, E=Eccentricity, $A_{Intrados}$=Number of blocks x block height

EQUILATERAL ARCH

- $x = R \cdot \cos\alpha - \dfrac{R}{2}$
- $y = R \cdot \sin\alpha$
- $C = 2R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $R = 2s$
- $r = R \cdot \sin 60°$
- $A_{Intrados} = \dfrac{\pi R \alpha}{180}$

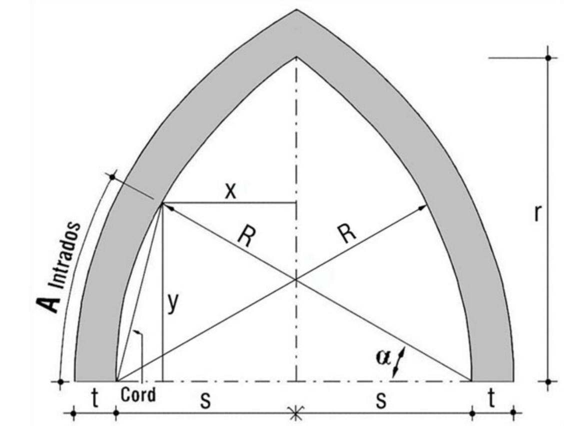


Fig. 172 – Equilateral arch

Where: R=Radius, r=Rise, s=1/2 span, t=Thickness, C=Cord, $A_{Intrados}$=Number of blocks x block height

EGYPTIAN ARCH

- $x = R_1 \cdot \cos\alpha - \dfrac{R_1}{2}$
- $y = R_1 \cdot \sin\alpha$
- $R_2 = \dfrac{3}{8} R_1$
- $C = 2R_1 \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R_1}$
- $A_{Intrados} = \dfrac{\pi R_1 \alpha}{180}$

Note:
x and y are calculated only for the radius R_1

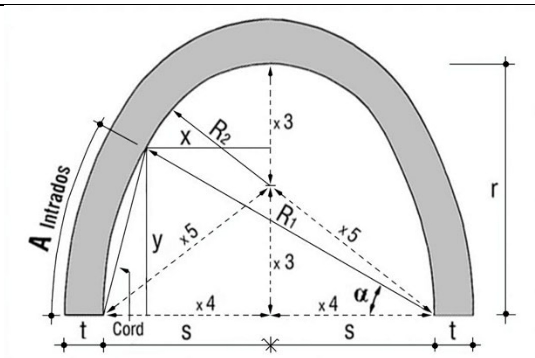


Fig. 173 – Egyptian arch

Where: R_1=First radius (centres on the springers), R_2=Second radius (centre on the axis), r=Rise, s=1/2 span, t=Thickness, C=Cord, $A_{Intrados}$=Number of blocks x block height

ELLIPTICAL ARCH

Equation of an ellipse: $\frac{4x^2}{D^2} + \frac{4y^2}{d^2} = 1$

- $D = T(\text{Thread})$
- $D = \sqrt{d^2 + 4f^2}$
- $D = \frac{2x}{\sqrt{1 - \frac{4y^2}{d^2}}}$
- $d = \sqrt{D^2 - 4f^2}$
- $d = \frac{2y}{\sqrt{1 - \frac{4x^2}{D^2}}}$
- $A_{Intrados} = \frac{\pi}{4}(D + d)$
- $f = \frac{\sqrt{D^2 - d^2}}{2}$
- $x = \frac{D}{2}\sqrt{1 - \frac{4y^2}{d^2}}$
- $y = \frac{d}{2}\sqrt{1 - \frac{4x^2}{D^2}}$
- $y = \sqrt{C^2 - \frac{D}{2} - x^2}$
- $Area_{Ellipse} = \frac{\pi}{4}Dd$

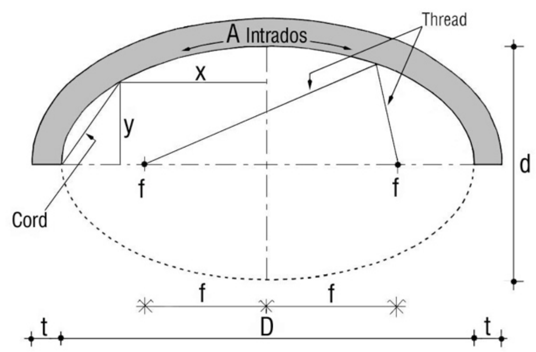


Fig. 174 – Elliptical arch

Where: D=Long diagonal, d=Short diagonal, T=Thread to trace the ellipse, f=Focal points
C=Cord, $A_{Intrados}$=Length of the arch inside

SEGMENTAL DOME

- $D = 2\sqrt{2Rr - r^2}$
- $r = R - \sqrt{R^2 - \frac{D^2}{4}}$
- $\sin\left(\frac{\alpha}{2}\right) = \frac{D}{2R}$
- $Area_{Inner} = 2\pi Rr$
- $Volume_{Inner} = \frac{\pi r^2}{3}(3R - r)$
- $Volume_{Shell} = \frac{\pi}{3}\left[(r + t)^2(3R + 3t - r) - r^2(3R - r)\right]$

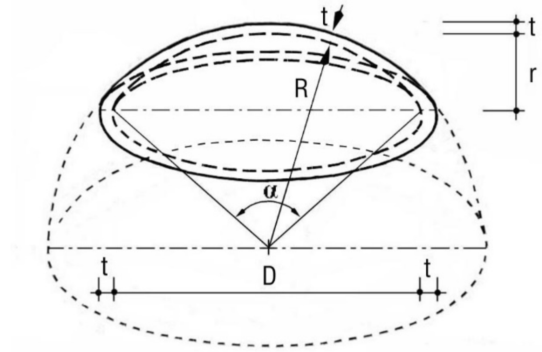


Fig. 175 – Segmental dome

Where: R=Inner radius of the actual sphere, D=Inner diagonal of the segmental sphere
r=Rise, t=Thickness, α=Inner angle of the dome

HEMISPHERICAL DOME

- $Area_{Inner} = 2\pi R^2$
- $Area_{Outer} = 2\pi(R + t)^2$
- $Volume_{Inner} = \frac{2\pi R^3}{3}$
- $Volume_{Shell} = \frac{2\pi t}{3}(3R^2 + t^2 + 3Rt)$

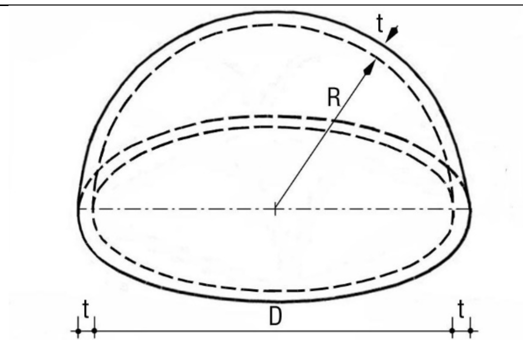


Fig. 176 – Hemispherical dome

Where: R=Radius, t=Thickness

DOME ON PENDENTIVES (SQUARE PLAN)

- $D = 2R = a\sqrt{2}$
- $d = a = \dfrac{D}{\sqrt{2}}$
- $r = \dfrac{a}{2}\left(\sqrt{2} - 1\right)$
- $Area_{Segmental\ sphere} = 2\pi Rr = \dfrac{\pi a^2}{2}\left(2 - \sqrt{2}\right)$
- $Area_{4\ Pendentives} = \pi a^2\left(\dfrac{3\sqrt{2}}{2} - 2\right)$
- $Area_{Entire\ dome} = \pi a^2\left(\sqrt{2} - 1\right)$

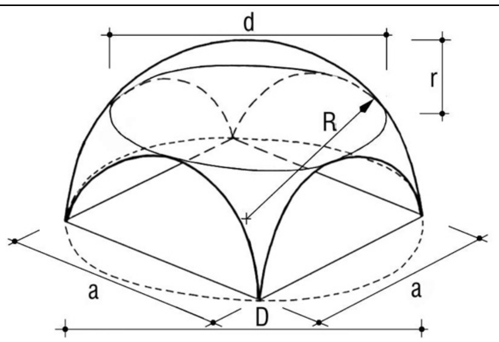


Fig. 177 – Dome on pendentives (Square plan)

Where: R=Radius, D=Diagonal of the hemisphere
d=Diagonal of the segmental sphere
r=Rise of the segmental sphere,
a=Side of the square

DOME ON PENDENTIVES (RECTANGULAR PLAN)

- $R = \dfrac{\sqrt{L^2 + \ell^2}}{2}$
- $D = \sqrt{L^2 + \ell^2}$
- $d = \ell$
- $r = R - \dfrac{L}{2} = \dfrac{\sqrt{L^2 + \ell^2} - L}{2}$
- $Area_{Segmental\ sphere} = 2\pi R\left(R - \dfrac{L}{2}\right)$
- $Area_{4\ Pendentives\ +\ zones} = 2\pi R\left(L + \dfrac{\ell}{2} - 2R\right)$
- $Area_{Entire\ dome} = 2\pi R\left(\dfrac{L}{2} + \dfrac{\ell}{2} - R\right)$

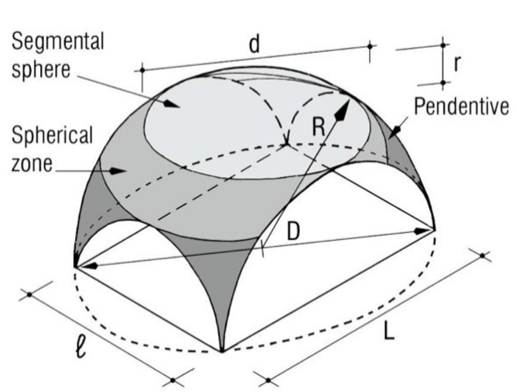


Fig. 178 – Dome on pendentives (Rectangular plan)

Where: R=Radius, r=Rise
L=Long side of the rectangle,
ℓ=Short side of the rectangle
D=Diagonal of the sphere / Rectangle,
d=Diagonal of the segmental sphere

DOME ON PENDENTIVES (HEXAGONAL PLAN)

- $R = b$
- $r = \dfrac{R}{2}$
- $d = a = 2R \cdot \cos 30^\circ$
- $Area_{Segmental\ sphere} = \pi R^2$
- $Area_{6\ Pendentives} = \pi R(3a - 5R)$
- $Area_{Entire\ dome} = \pi R(3a - 4R)$

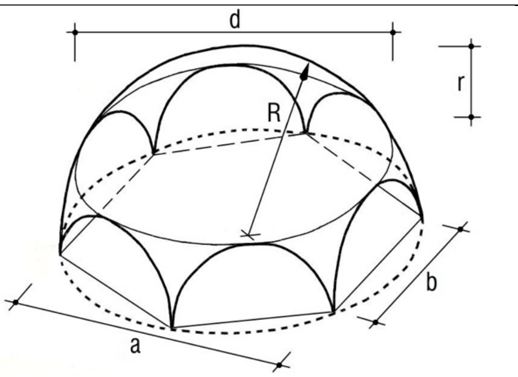


Fig. 179 – Dome on pendentives (Hexagonal plan)

Where: a=Diagonal inscribed to the hexagon
b=Side of the hexagon, R=Radius, r=Rise
d=Diagonal of the segmental sphere = Inscribed to the hexagon

DOME ON PENDENTIVES (OCTAGONAL PLAN)

- $R = \frac{b}{\sqrt{2}}\sqrt{\left(2+\sqrt{2}\right)}$
- $r = R - \frac{b}{2}$
- $r = R\left(1-\sin 22.5^\circ\right)$
- $a = b\left(1+\sqrt{2}\right)$
- $Area_{Segmental\ sphere} = 2\pi R^2\left(1-\sin 22.5^\circ\right)$
- $Area_{8\ Pendentives} = 2\pi R^2\left(4\cos 22.5^\circ + \sin 22.5^\circ - 4\right)$
- $Area_{Entire\ dome} = 2\pi R^2\left(4\cos 22.5^\circ - 3\right)$
- $d = a = 2R \cdot \cos 22.5^\circ$
- $D = \frac{a}{\cos 22.5^\circ} = \frac{b}{\sin 22.5^\circ}$
- $b = \frac{a}{\left(1+\sqrt{2}\right)}$

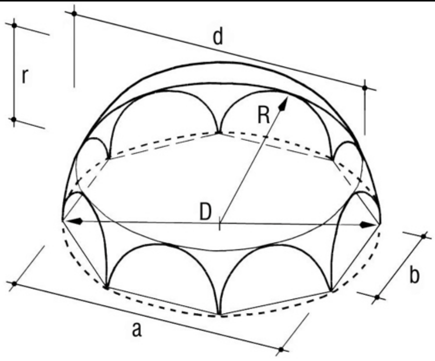


Fig. 180 – Dome on pendentives (Octagonal plan)

Where: a=Diagonal inscribed to the octagon, b=Side of the octagon, R=Radius, r=Rise
d=Diagonal of the segmental sphere = Inscribed to the octagon
D=Diagonal of the hemisphere =circumscribed to the octagon

GROIN DOME (SQUARE PLAN)

- $D_{Vault} = a = 2R$
- $D_{Groin} = D_{Vault} \cdot \sqrt{2}$
- Point at y $\Rightarrow$ $X = x\sqrt{2}$
- Area $= \pi a^2$
- $r = R$
- $f = R$
- $d = 2R$

Where: R=Radius of the semicircular vault
r=Rise, a=Side of the square
D Vault=Diagonal of the semicircular vault
d=Short diagonal of the ellipse
f=Focal points of the ellipse (groin)
D Groin=Long diagonal of the ellipse
=Diagonal of the groin

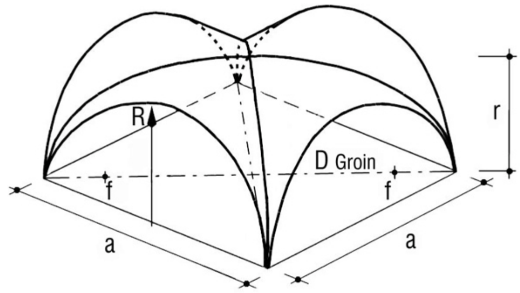


Fig. 181 – Groin dome (square plan)

Note that the groin is an ellipse which can be calculated *(See Geometric Formulas, Elliptical Arch)* or traced out graphically as shown below.

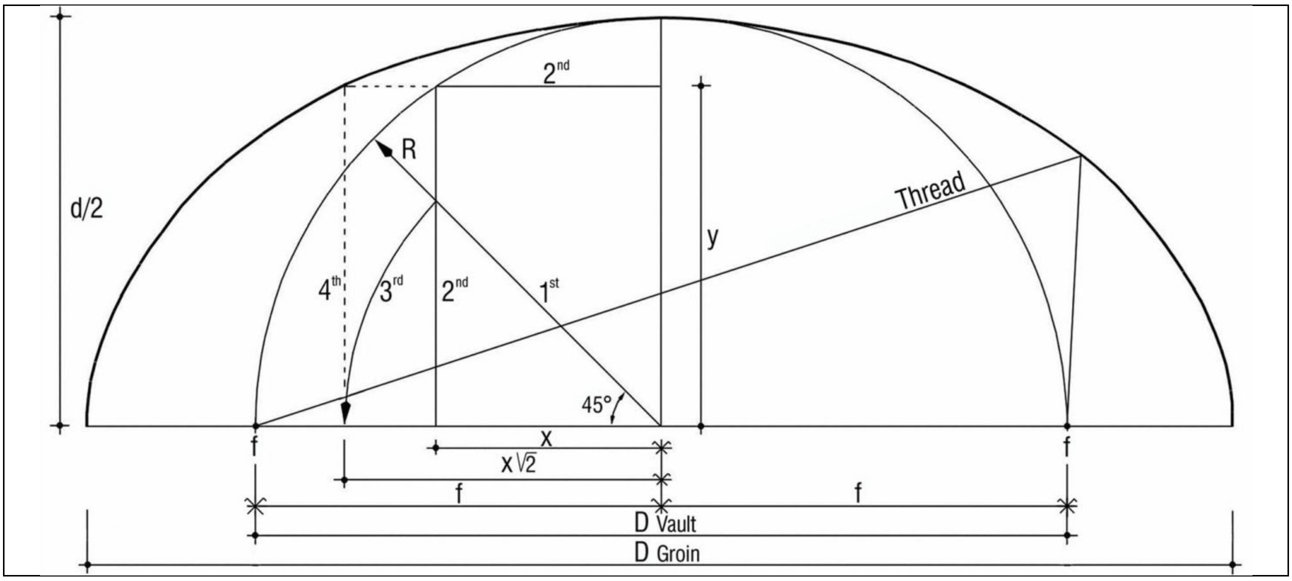


Fig. 182 – Tracing the elliptical groin from the semicircular vault

CONICAL DOME

- $\text{Area} = \pi R \sqrt{R^2 + h^2}$
- $\text{Volume} = \dfrac{\pi h R^2}{3}$
- $\text{Volume}_{\text{Shell}} = \dfrac{\pi h}{3}\left(2Rt + t^2\right)$

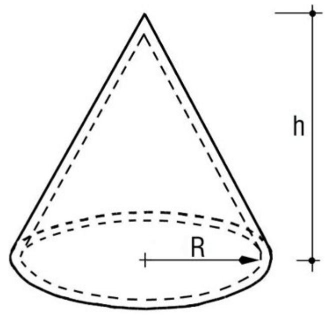


Fig. 183 – Conical dome

Where: R=Radius, h=Height, CG=Centre of gravity
t=Thickness

SPHERICAL ZONE

- $y = \sqrt{R^2 - \dfrac{D^2}{4}}$
- $D = 2\sqrt{R^2 - y^2}$
- $d = 2\sqrt{R^2 - (y + r)^2}$
- $\text{Area}_{\text{Inner}} = 2\pi Rr$
- $\text{Volume}_{\text{Inner}} = \dfrac{\pi r}{24}\left(3D^2 + 3d^2 + 4r^2\right)$

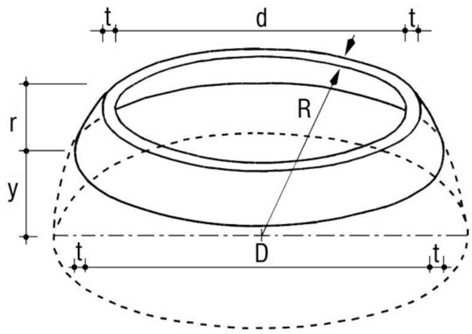


Fig. 184 – Spherical zone

Where: R=Radius of the sphere, r=Rise,
d=Upper diagonal of the zone
D=Lower diagonal of the zone

SEGMENTAL SECTOR

- $R = \dfrac{r^2 + s^2}{2r}$
- $r = R - \sqrt{R^2 - s^2}$
- $s = R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $y = \dfrac{8s^4}{r\left(3r^2 + 16s^2\right)}$
- $\sin\left(\dfrac{\alpha}{2}\right) = \dfrac{s}{R}$
- $\alpha = \dfrac{180\, A_{\text{Arch}}}{\pi R}$
- $\text{Area} = \dfrac{r\left(3r^2 + 16s^2\right)}{12s}$
- $A_{\text{Arch}} = \dfrac{\pi R \alpha}{180}$

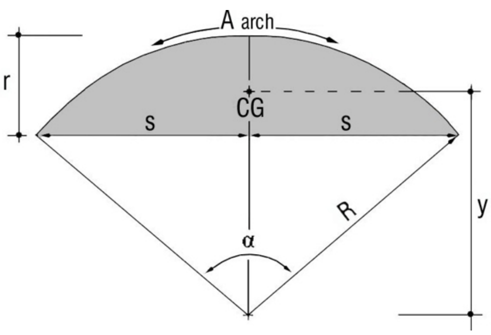


Fig. 185 – Segmental sector

Where: R=Radius, r=Rise, s=1/2 span
A_{Arch}=Length of the arch, CG=Centre of gravity

CIRCULAR SECTOR

- $R = \dfrac{r^2 + s^2}{2r}$
- $r = R - \sqrt{R^2 - s^2}$
- $s = R \cdot \sin\left(\dfrac{\alpha}{2}\right)$
- $y = \dfrac{4Rs}{3A_{Arch}} = \dfrac{240s}{\pi\alpha}$
- $\alpha = \dfrac{180\,A_{Arch}}{\pi R}$
- $\sin\left(\dfrac{\alpha}{2}\right) = \dfrac{s}{R}$
- $A_{Arch} = \dfrac{\pi R \alpha}{180}$
- $Area_{Sector} = \dfrac{\pi \alpha R^2}{360}$

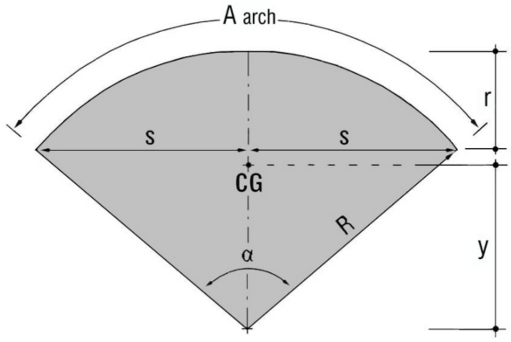


Fig. 186 – Circular sector

Where: R=Radius, r=Rise, s=1/2 span CG=Centre of gravity, A_{Arch}=Length of the arch

CIRCULAR SEGMENT

- $A_{Intrados} = \dfrac{\pi R \alpha}{180}$
- $A_{Extrados} = \dfrac{\pi (R + t)\alpha}{180}$
- $\sin\left(\dfrac{\alpha}{2}\right) = \dfrac{s}{R}$
- $\alpha = \dfrac{180 \cdot A_{Intrados}}{\pi R}$
- $x = \left(\dfrac{2}{3}\left[\dfrac{(R+t)^3 - R^3}{(R+t)^2 - R^2}\right]\dfrac{360s}{\pi\alpha(R+t)}\right) - R$
- $Area_{Segment} = \dfrac{\pi \alpha t (t + 2R)}{360}$

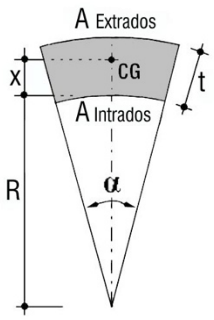


Fig. 187 – Circular segment

Where: R=Radius, t=Thickness, $A_{Intrados}$=Length of the arch inside, $A_{Extrados}$=Length of the arch outside

CENTRE OF GRAVITY OF A SEGMENT (TRAPEZOID)

When circular segments are very short and have little curvature, e.g. voussoirs, they can be approximated as trapezoids. The calculation for the CG of a circular segment requires more calculation than for a trapezoid *(See Geometric Formulas, Circular Segment)*, and, as the location of the CG is very similar if the segment is short, it is considered an appropriate approximation to analyse a segment as a trapezoid.

The CG is located on the median of the different sides and at *x* from the intrados.

$$x = \frac{t}{3}\frac{(Ext + 2Int)}{(Ext + Int)}$$

Where:
- t = Thickness
- me = Median of the segment
- Ext = Length of the segment extrados
- Int = Length of the segment intrados

The location of the CG can be approximated to the intersection of the medians.

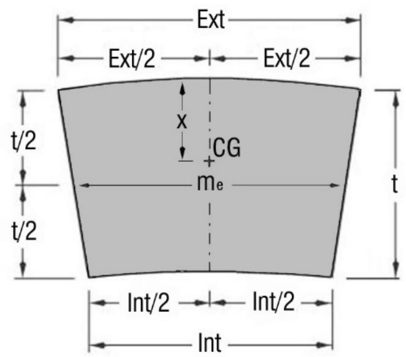


Fig. 188 – Centre of gravity of a segment (analyzed as a trapezoid)

5.3 GLOSSARY

NOTES

The list of terms presented below is not an exhaustive one. The following terms are explained only with respect to the meaning and relevance they have in the context of this manual. The words in brackets are French translations.

Abutment *(Piédroit)*
A pier, such as a buttress or a wing wall of solid masonry, which is erected to counter the thrust of an arch, vault or dome.

Annular vault *(Voûte annulaire)*
A vault that curves in plan, such as the annular barrel vault that covered many an early ambulatory in European religious buildings.

Arcade *(Arcade)*
A range of arches resting on piers or column. A blind arcade is an arcade attached to a wall.

Arch *(Arc)*
A curved structure spanning an opening and made of wedge-shaped stones or blocks, which support each other by compression.

Banded barrel vault *(Voûte en berceau nervurée)*
A barrel vault whose semi-cylindrical shape is stiffened at intervals by previously built transverse arches that project beneath its intrados.

Barrel vault *(Voûte en berceau)*
The simplest of all vault shapes, basically semi-cylindrical in shape, and therefore everywhere semicircular in cross section. Sometimes barrel vaults have a slightly pointed section.

Bed – Of a masonry course *(Lit – Assise de maçonnerie)*
In masonry, the joint at the base of a row or course of stones or bricks. It is also the surface on which the building units are laid or bedded. Normally it is horizontal, but in arches, vaults and domes the beds are tilted with a rising angle towards the top of the structure.

Bond pattern
An arrangement of masonry units to create a cohesive wall. This arrangement shall be such that the odd and even courses do not coincide with each other, creating a staggered pattern.

Block faces (Bed, Header, Laying, Stretcher)

- **Bed face**: The horizontal bottom side of the block which is laid on the mortar.
  Note that frogs shall be on the bed face and not the laying face.
- **Header**: The short side of the block.
- **Laying face**: The horizontal top side of the block on which the mortar is laid.
- **Stretcher**: The long side of a block.

Bucket or Basket handle arch *(Arc en anse de panier)*
An arch composed of 3 or more centres: 2 centres are located on the springer line, symmetrical to the axis. 1 centre is located on the axis, below the springer line. Bucket arches can have 5, 7 or more centres (always an odd number). An ellipse can be considered as a bucket arch, which has an infinite number of centres.

Buttress ***(Contrefort)***
A rectangular column of masonry typically either integrated into a wall in its entirety, or integrated into the wall only at its base. It can also be free-standing. It receives and balances the lateral thrust of a vault.

Catenary Arch ***(Arc en chaînette)***
The curve assumed by a freely suspended chain or flexible cable under the action of gravity. The centre line of the links is the line of tensile stress.
In an arch which has the shape on an inverted catenary curve, the voussoirs correspond to the links of the chain. Just as the links of the chain are only in tension, the voussoirs of the inverted catenary arch are only in compression. The curve taken by the chain corresponds to the line of thrust of the arch. Catenary arches are always the most stable and their thickness can be minimized.

Centring ***(Cintre)***
Wooden or steel support for building an arch or vault, which is removed after completion.

Clerestory ***(Lanterneau)***
The upper part of the nave, choir and transepts, containing a series of windows above the roofs of the aisles.

Cloister Vault or Cloister Dome ***(Voûte ou dôme en arc de cloître)***
A four-sided dome, which consists of portions of the surfaces of two barrel vaults, often pointed, that intersect perpendicularly. The diagonal arches of the cloister dome are identical to the groins of the groin vault, which have the same generating geometry. The cloister dome does not show the cross section of the barrel vault. The thrusts are continuous along the entire perimeter of the supporting walls, rather than confined to the corner piers, as in the groin vault.

Compressed Stabilised Earth Block (CSEB)
A masonry unit which is manufactured by mixing suitable soil, cement, water and sometimes sand in correct quantities. The mix is compressed straight away in a either a manually operated or mechanized press.

Compressive strength, σ_C
The average stress under which three or more blocks crush in a testing machine. The compressive strength, expressed in Mega Pascal (MPa), shall be tested in accordance to the relevant standard. Note that the compressive strength is not the admissible strength, but rather the failure stress of blocks. The compressive strength shall be tested under dry condition (σ_{Cd}) and wet conditions, after 24 hours immersion, (σ_{Cw}).

Corbel ***(Corbeau)***
A block of stone projecting from a wall, which supports a roof, a vault, a parapet, a shaft or any other feature.

Corbelling ***(Encorbellement)***
A structural pattern for reducing a given span by projecting out in successive courses. The projection of each course over the other should preferably be 1/4 of the block length, in order to have a good bond pattern.

Course ***(Assise)***
A single layer of masonry units of the same height, including the bed joint.

Crown of arch or vault ***(Sommet de l'arc ou voûte)***
The highest part of the curve of an arch or vault, whether pointed or not.

Curing
The process of keeping the blocks in a humid state for some time, to allow for the hardening process of the stabiliser. Blocks shall not dry during the curing period, which varies according to the stabiliser:
- 4 weeks for cement stabilised blocks.
- 2 to 4 weeks or lime stabilised blocks, depending on the soil and lime quality.

Dead load ***(Charge propre)***
The weight of the structure itself (self-weight and any overloading).

Decentring ***(Décoffrage)***
The action of removing the centring. Centring must be slid down slowly, as the wedges are pulled out progressively and symmetrically.

Design strength
The compressive crushing strength of the block multiplied by a partial safety factor.

Diagonal or Intersecting ribs ***(Croisée d'Ogive)***
The diagonal ribs which appear at the intersection of two pointed vaults in the Gothic system of vaulting (i.e. a groin vault with the generating geometry of a pointed arch).

Discharging or Relieving arch ***(Arc de décharge)***
An arch built into the masonry of a wall to relieve downward pressure above a lintel or an opening beneath.

Dome ***(Dôme)***
A vaulted construction spanning a circular or polygonal area. A circular dome is generated by the rotation of its cross section around a central axis. Square or polygonal domes are generated by the intersection of vaults.

Earth
See 'Soil'

Egyptian arch ***(Arc Egyptien)***
An arch named as such because it has certain proportions which were used extensively in Egypt. These proportions are based on the Pythagorean triangle 3, 4, 5.

Equilateral arch ***(Arc en tiers points)***
A pointed arch which has its geometric centres just on the inner edge of the springer. Thus, its radiuses and span have the same dimension.

Extrados ***(Extrados)***
The upper or convex surface of an arch, vault or dome.

Faceted dome ***(Dôme à facettes)***
A dome resting on a polygonal plan, which is generated by the intersection of vaults springing from 2 opposite sides of the polygon.

Fan vault ***(Voûte en éventail)***
A conical type of vault in which the length and curvature of all the ribs (which are mainly decorative and not structural) is similar. It was used during the English Perpendicular Gothic period.

Flying buttress ***(Arc boutant)***
The masonry strut from clerestory wall to buttress top. The flying buttress is normally built as a rampant arch and the top of the buttress is loaded with masonry, often in the shape of a pinnacle.

Force diagram or Funicular polygon
Scaled diagram that represents the forces in an arch. From this diagram, the magnitude of the Weight of the arch (W), Horizontal Thrust (HT) and Thrust (T) can be determined.

Form diagram
Cross section of the arch (sometimes only ½ of the arch), which is used for the purpose of funicular analysis.

Formwork *(Coffrage)*
In the construction of arches and vaults, that portion of a false work structure by which the curve is given its shape and is supported while it is being built. It is also called a centring. Arches always need a formwork to be built. Vaults can be built without formwork, with the free spanning technique or one of several other formworkless construction techniques.

Free Spanning technique *(Technique autoportante)*
A method developed at the Auroville Earth Institute to build vaults of any shape and size without support. Compressed stabilised earth blocks are laid with soil cement stabilised glue, either with vertical or horizontal courses. This technique is a development of the Nubian vaulting technique.

Funicular geometry
The most efficient geometry for a given loading.

Groin *(Arête de voûte)*
The solid, continuous inner angle or curved intersection formed by the meeting of two simple vaults that cross each other at any angle.

Groin vault or Groin dome *(Voûte d'arête)*
A vaulted structure created by the intersection (most commonly perpendicular) of two tunnel or barrel vaults. In a simple groin vault over a square bay, seen from below, the groins are the salient ridges that arch across over the diagonals of the square. Groin vaults were one of the typical architectural features of the Roman style in Europe.

Grout *(Coulis, Barbotine)*
A rich binder (with no large aggregate) of very fluid consistency which can be poured into the joints of masonry.

Haunch *(Rein)*
The part of an arch or vault directly above the springer. Depending on the arch shape, the haunch will rise 1/4 to 1/3 of the curve above the springer.

Hoop Force
The hoop force is the result of compression forces acting circumferentially in the dome.

Horizontal Thrust (HT)
The horizontal component force of Thrust (T), HT, is the same magnitude throughout an arch and is determined by the geometry of the arch.

Intrados *(Intrados)*
The concave or inner surface of an arch, vault or dome.

Jack arch *(Voûtain)*
A narrow and very flat segmental vault, which spans between beams. Jack arches placed in sequence are used for floor systems.

Joint
Horizontal or vertical space between two masonry units which is typically filled with mortar.

Joint – Bed joint
The horizontal mortar joint of a course in a wall.

Keystone *(Clé de voûte)*
The central or topmost voussoir in an arch; it is also known as the key. Keying, or keying in, is the act of inserting the final voussoir at the crown of an arch or vault. For segmental or semicircular arches, the keystone is often a voussoir like any other. In the case of a pointed arch, the keystone has often a particular shape, feature and size.

Line of Thrust (LT) ***(Courbe de pressions)***
A theoretical line which represents the successive pushing action as weight is transferred down through an arch, vault or dome. LT is the locus of the intersection of internal resultant forces acting in the arch. The trajectory and position of this line determines the stability of the masonry arch. LT should always remain in the middle third of the arch section and the pier for stable and safe.

Live load ***(Surcharge de service)***
The weight of any transient feature, intermittent force, or movable body that is not part of the permanent structure.

Load bearing wall
Wall primarily designed to carry the vertical load of the building, its own weight and the live load.

Load – Dead loads
The weights of all materials for walls, floors, roofs, ceilings, stairways, built-in partitions, finishes, cladding and other items which are part of a building. All permanent loads are considered dead loads.

Load – Live loads
Live loads include all the forces that are variable within and on the building, such as weight of inhabitants, furniture and the environmental loads: wind load, snow load, rain load, earthquake load, flood load.

Load line
The vertical line in the force diagram which represents the total external loads of an arch (typically only self-weight).

Masonry
A heterogeneous assemblage of masonry units, either laid *in situ* or constructed with prefabricated components, in which the masonry units are bonded solidly together with mortar or grout.

Masonry unit
A building component, generally of a small size, which is used for building walls or other parts of a building. Masonry unit can be CSEB, concrete block, fired bricks, stones, etc.

Mega Pascal, MPa
A Mega Pascal is 10^6 Pascal. Pascal, (symbol Pa) is the SI derived unit of stress.
1 Pa = 1 Newton/m^2 and 1 MPa = 10.19716 kg/cm^2

Mortar
A workable paste used to bind masonry units together and fill the gaps between them. Mortar is composed of aggregates (sand and/ or soil), a binder (usually cement or lime) and water.

Mortar – Stabilised earth mortar
Mortar of cement, soil and sand which are mixed in various proportions to meet the particular requirements. A composition of stabilised earth mortar shall be represented as SEM 1: 4: 8 (1 cement, 4 soil and 8 sand).

Nave ***(Nef)***
The main body, or middle part, length wise, of a church interior, extending from the principal entrance to the choir.

Nubian Technique ***(Technique Nubienne)***
A method to build vaults and domes, which was developed in Nubia, south of Egypt. Adobes (sun dried mud bricks) are laid with a clayey mortar and without support. The vault is built with leaning courses, arch after arch. This technique can be used to build any shape and size of vault. It is also used to build domes, traditionally hemispherical in Egypt.

Oculus *(Oculus)*
The circular opening which sometimes exists at the top of a dome. An oculus may be of any size, since a true dome is stable at the completion of any horizontal ring.

Pendentive *(Pendentif)*
One of the triangular segments of the lower part of a hemispherical dome, used to create the transition at the angles from a square or polygonal base below to a circle above. The top junction of the pendentives will be circular and will be the first ring on which a segmental or hemispherical dome may rest.

Pier *(Contrefort, Trumeau)*
The solid support from which an arch spring. A masonry support designed to sustain vertical load. Piers may be simple (round, square, rectangular) of compound, composite, multiform and of more complex profile.

Pointed arch *(Arc ogival)*
An arch formed of two arches of equal radius, which are located symmetrically on either side of the symmetry axis.

Rampant or Ramping arch *(Arc rampant)*
An arch that spans between springers located at different levels.

Rib *(Nervure)*
An arch of thin section in a vault. One of the salient stone arches that visually divide a Gothic vault into compartments.

Rib vault or Ribbed vault *(Voûte nervurée)*
The characteristically Gothic system of vaulting, in which independent ribs are first constructed and then the thin web of the vault proper is built in the panels or compartments they define.

Ring beam *(Chaînage)*
A tie beam, circular, square or rectangular, which is meant to neutralize the thrust of a dome.

Rise *(Flèche)*
The vertical distance, in an arch, vault or dome, from springer to the crown intrados.

Safety factor
A coefficient considered for calculating safely a building.

Segmental arch *(Arc surbaissé)*
An arch which has its centre point lower than the springer line. It is less than half a circle.

Segmental dome *(Calotte sphérique)*
The upper part of a circular dome (i.e. In the case of a hemispherical dome on pendentives). In the case of a dome starting from a circle, the centre of a segmental dome is lower than the springer line.

Shrinkage
Stress induced by shrinkage of clay in the material and creates cracks in a wall. This effect can be reduced by modifying mortar proportions or decreasing water.

Soil or Earth
Soils are the result of transformation of the parent rock under the influence of a range of physical, chemical and biological processes related to biological and climatic conditions and to animal and plant life. Soils may or may not contain organic matter, and are comprised of gravels, sands, silts and clays.

Span ***(Portée)***
The distance between the springers of an arch or a vault. The diagonal of a dome built on a circular plan. The distance between two opposite walls of a square or polygonal dome.

Spandrel ***(Tympan)***
The wall surface outside of the arch and approximately triangular, between the extrados of the arch, the vertical line passing through the springer and the horizontal line touching the crown extrados.

Springer ***(Sommier)***
The stone or block supporting the arch, or vault. Depending on the arch type it can be horizontal or with an angle.

Springer line ***(Ligne de naissance)***
The theoretical line connecting springer to springer, at the intrados of the curve (where span is measured).

Squinche ***(Trompe d'angle)***
A half-conical niche placed in the corners of a rectangular structure to form the base for the support of a dome or polygonal vault.

Template ***(Gabarit)***
A lightweight guide used to indicate the shape of an arch. It can be made of any material and dimensioned either to the intrados or extrados curve. It is not meant to support any load, but only to define the shape of the arch.

Tension ties ***(Tirant)***
A tie, often made of steel, which links the springers. It is designed to neutralize the thrust of an arch or vault.

Thrust (T) ***(Poussée)***
The force which pushes downwards and outwards through an arch, vault or dome. The thrust is composed of two forces: the weight of the arch and the horizontal thrust. The latter can be neutralized by various ways: all kinds of abutments, tension ties or ring beams. The thrust is always applied on the springer with an angle.

Tunnel vault ***(Voûte en tunnel)***
A longitudinal arched tunnel, the simplest kind of vault in terms of its three-dimensional form.

Vault ***(Voûte)***
An arched ceiling of masonry. A basic vault is created by an arch cross section which is slid longitudinally on the springer walls (an extrusion in one direction).

Voussoir ***(Voussoir)***
One of the wedge-shaped blocks that makes up an arch or vault.
When a voussoir has the shape of a parallelepiped brick, the wedge-shape is given by a triangular mortar joint.

Wedge ***(Cale, Coin)***
A piece of wood or any other material, thick at one end and tapered to the other end. Wedges are used to support the centring and to allow easy decentring by slipping them out from under the formwork.

Weight of the Masonry (W)
The vertical component force of Thrust, W, represents the weight of the arch and is determined by the total volume and density of the designed arch.

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